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Vibration tests of 5-storey steel frame with viscoelastic dampers

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Abstract

This paper presents a design process for viscoelastic dampers and experimental test results of a 5-storey single bay steel structure with added viscoelastic dampers. The mechanical properties of viscoelastic dampers and the dynamic characteristics of the model structure were obtained from experiments using harmonic excitation, and the results were used in the design process. The additional damping ratios required to reduce the maximum response of the structure to a desired level were obtained first. Then the size of dampers to realize the required damping ratio was determined using the modal strain energy method by observing the change in modal damping ratio due to the change in damper stiffness. The designed viscoelastic dampers were installed in the first and the second stories of the model structure. The results from experiments using harmonic and band limited random noise indicated that after the dampers were installed the dynamic response of the full-scale model structure reduced as desired in the design process.

Keywords: Viscoelastic dampers; Full-scale test; Vibration control; Modal strain energy method

1. Introduction

Viscoelastic dampers (VED) are highly effective in mitigating the dynamic responses of building structures due to wind or seismic excitation, and practically were used in the twin towers of the World Trade Center in New York for the reduction of wind-induced vibrations. Many researches have been conducted to derive analytical models for VED and to verify the effect on structural control through experiments [1–4]. Previous studies showed that VED can increase structural damping significantly, which brings the decrease of structural responses, such as displacement and absolute acceleration. The results of these studies also indicated that the performance of VED depends on factors such as excitation frequency and environmental temperature, and so the effects of VED should be evaluated by considering these factors. However, in most experiments, small-scale model structures with scaled VED were used, and only few experimental researches were conducted which verified the effectiveness of VED in a full-scale model structure.

Another issue related to practical applications of VED is the development of the design procedure for VED to achieve a prescribed structural response level. For this purpose, a systematical decision of such design factors as number, size, optimal location, and installation method is required. In many studies, however, VED of arbitrary sizes were installed and their vibration control effect was observed. Zhang et al. proposed a sequential procedure for optimally placing VED based on the concept of degree of controllability [5]. Lee et al. determined the location and the size of VED by using pole assignment method [6]. These studies showed that placing VED at the position with the largest inter-storey displacement is most effective. Chang et al. presented a design guideline for steel frame structures with VED, based on the results of previous studies, and verified the effectiveness of the design procedure by conducting a experiment for 2/5-scale structural model [7]. Although this design procedure provides detailed prescription and its effec-

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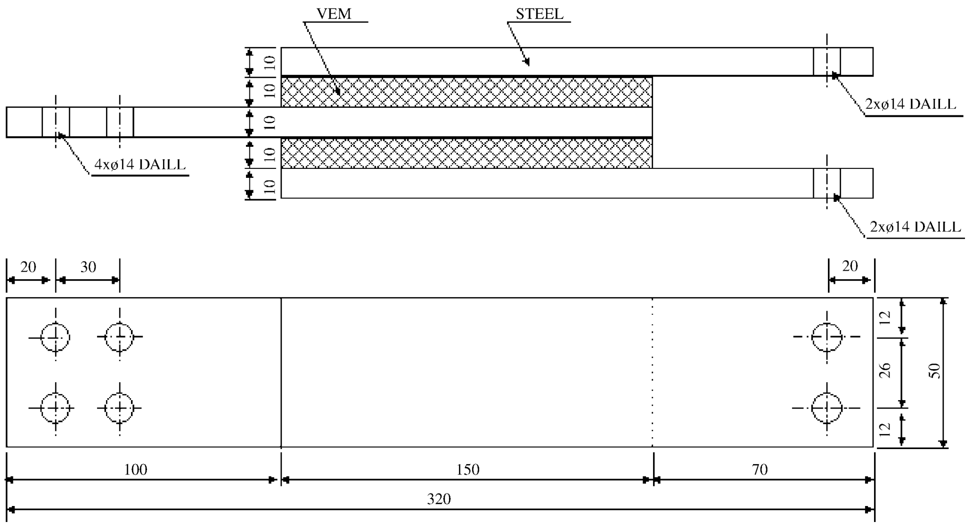

Fig. 1. Test specimens of viscoelastic materials.

tiveness is verified through experimental study, further experimental studies are required for practical application. Especially, through full-scale experiment, many design considerations can be provided and the results will be a basis for structural application of VED.

In this study, based on the results of cyclic tests of damper elements and on the results of system identification tests of a 5-storey full-scale model structure vibrated using the hybrid mass driver (HMD) located on the fifth floor, VED were designed to satisfy a given target response. The additional damping ratios required to reduce the maximum dynamic response of the structure to a given level were obtained first by convex model [8]. The size of dampers was determined using the modal strain energy method which uses the required modal damping ratio [7]. The stiffness of the supporting braces was also considered in the determination of the modal properties. Then the structure with the VED installed in the lower two stories which have the largest inter-storey displacement was tested to validate their vibration control effect. The structure was excited by the HMD which generates sinusoidal and band-limited excitation forces. Only linear-elastic response was considered and soil-structure interaction and torsional response were neglected.

2. Properties of viscoelastic materials

The design of VED requires the knowledge of the mechanical properties of viscoelastic material. The properties of the viscoelastic material depend on variables such as temperature, excitation frequency, and strain, and therefore, the effect of these variables on the mechanical properties of VED were investigated by experiments. Test specimens shown in Fig. 1 were made of two layers of viscoelastic materials, each layer 10 mm in thickness and 150 cm^2 in area. Ten cyclic tests were conducted on the specimens by using sinusoidal excitation with 0.5 Hz at the temperature of 24 °C with various maximum strains of 20, 25, 50, 75, and 100%. Table 1 presents the test results for storage stiffness and loss factor obtained for different strains, and shows that the loss factor was not sensitive to the maximum strain. In addition, ten cyclic tests were also conducted by using sinusoidal excitation at 24 °C with various forcing frequencies and maximum strains. Test results shown in Table 2 indicate that storage stiffness and loss factor significantly depend on excitation frequency. Finally temperature-dependent behavior was investigated by a series of tests at various temperatures with excitation frequency of 0.5 Hz and strains of 20 and 50%. Table 3 shows that the loss factor is very sensitive to temperature. Fig. 2 shows that the shear storage modulus increases with increasing excitation frequency.

3. Experimental setup

3.1. Model structure for experiment

The experimental model, which is shown in Fig. 3, is a full-scale five-storey steel structure with storey height

Table 1
Storage stiffness and loss factors for viscoelastic test specimen shown in Fig. 1 for different displacements (0.5 Hz harmonic test)

Horizontal displacement	Storage stiffness (kN/m)	Loss factor
0.20 *t* (2.0 mm)	1260	0.66
0.25 *t* (2.5 mm)	1203	0.67
0.50 *t* (5.0 mm)	1080	0.70
0.75 *t* (7.5 mm)	966	0.72
1.00 *t* (10.0 mm)	926	0.69

t is thickness at 10 mm.

Table 2
Storage stiffness and loss factors for viscoelastic test specimen shown in Fig. 1 for different frequencies

Horizontal displacement	Excitation frequency	Storage stiffness (kN/m)	Loss factor
0.20 t (2.0 mm) (24 °C)	0.01 Hz	492	0.27
	0.1 Hz	709	0.41
	0.2 Hz	847	0.48
	0.3 Hz	932	0.54
	0.5 Hz	1102	0.67
	1.0 Hz	1502	0.73
	2.0 Hz	1718	0.90
	3.0 Hz	1760	1.08
	4.0 Hz	2158	1.02
	5.0 Hz	2364	1.04
	6.0 Hz	2248	1.15
0.50 t (5.0 mm) (24 °C)	0.01 Hz	472	0.28
	0.1 Hz	718	0.46
	0.2 Hz	856	0.55
	0.3 Hz	940	0.61
	0.5 Hz	1121	0.68
	1.0 Hz	1370	0.80
	2.0 Hz	1539	1.01
	3.0 Hz	1878	1.01
	4.0 Hz	1930	1.13
	5.0 Hz	2286	1.19
	6.0 Hz	2204	1.30
1.00 t (10.0 mm) (24 °C)	0.01 Hz	470	0.32
	0.1 Hz	674	0.52
	0.2 Hz	771	0.60
	0.3 Hz	886	0.61
	0.5 Hz	968	0.70
	1.0 Hz	1085	0.81
	2.0 Hz	1327	0.85

t is thickness at 10 mm.

of 6 m, plan of 6 × 6 m, and storey mass of 20 ton. Each floor is composed of four identical wide-flange type steel columns. A HMD shown in Fig. 4, is installed on the fifth floor to excite the model structure. Fig. 5 shows the location of VED and accelerometers.

Table 3
Storage stiffness and loss factors for viscoelastic test specimen shown in Fig. 1 for different temperatures (0.5 Hz harmonic test)

Horizontal displacement	Temperature (°C)	Storage stiffness (kN/m)	Loss factor
0.20 t (2.0 mm)	+40	637	0.39
	+30	833	0.47
	+24	1260	0.66
	+10	2411	0.97
	0	4478	0.97
	−5	10,069	0.79
	−10	20,466	0.47
0.50 t (5.0 mm)	+40	553	0.40
	+30	696	0.53
	+24	1080	0.70
	+10	2032	0.94
	0	3711	1.04
	−5	6471	1.06
	−10	14,619	0.58

t is thickness at 10 mm.

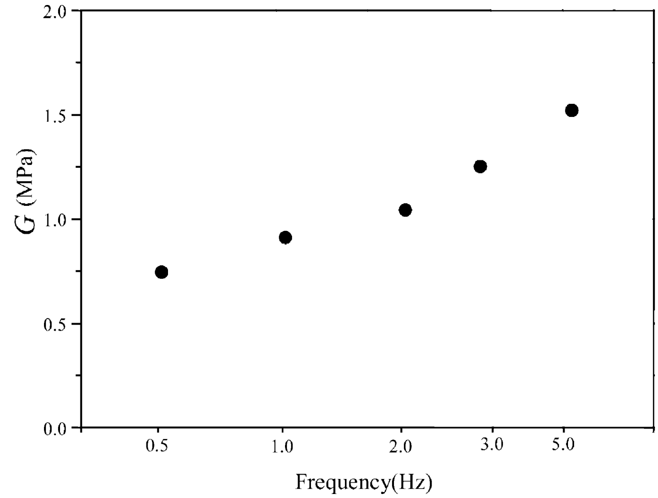

Fig. 2. Shear storage modulus of viscoelastic materials shown in Fig. 1 with 50% strain.

3.2. System identification

The fundamental frequency was found experimentally to be 0.50Hz by investigating the free vibration response after the excitation by the HMD stops. Since each storey mass was known and storey stiffness was uniformly distributed, the storey stiffness could be estimated, which was 2440 kN/m. Then, eigenvalue analysis was performed for the full-scale structure. For the analysis, the structure was assumed to be a shear building with 5 DOF, and that the weight of each storey was concentrated on the floor. Five natural frequencies were identified from eigenvalue analysis: 0.51, 1.46,

Fig. 3. Full-scale model structure for experiment.

Fig. 4. Photograph of the HMD used in the experiment experiment.

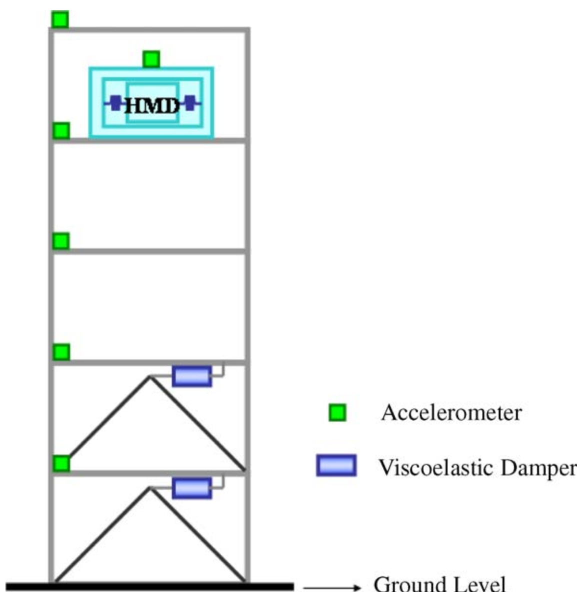

Fig. 5. Locations of VED and accelerometers.

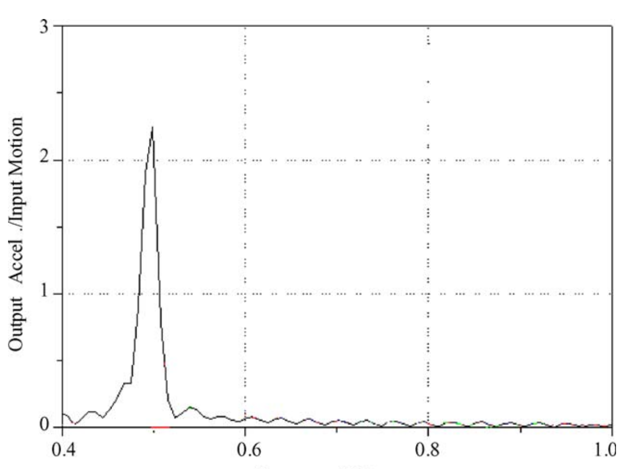

Fig. 6. Transfer function from the acceleration of HMD to the acceleration of the roof storey for the uncontrolled structure.

2.30, 2.95, and 3.37 Hz. To obtain the modal damping ratio experimentally, sinusoidal loadings with frequency ranges from 0.4 to 0.8 Hz were applied to the structure at the increment of 0.05 Hz. Near the predicted natural frequency the increment was further reduced. Fig. 6 plots the transfer function for the acceleration of the top storey, from which the fundamental natural frequency of 0.50 Hz was observed again and the corresponding damping ratio of 1.98% was obtained by using half power bandwidth method. In the case of seismic analysis, modal mass participation factors in terms of percentage of the 100 metric ton total mass for first to fifth mode are, respectively, 87.95, 8.72, 2.42, 0.75, and 0.16%, which indicates that the first mode predominates over other modes.

4. Determination of required damping ratio

To design the VED to be used in experiment, we first identified the fundamental vibration mode and the modal damping ratio, assuming that the structural response is dominated by the fundamental mode. Then the damping ratio of the structure required to achieve a given target response was computed using the convex model, which is one of the methods of predicting maximum responses for non-stationary earthquake loads. The convex model is known to be useful especially when only limited amount of information about load exists, although the solution of the model tends to be conservative. Many variables can be used to represent the uncertainty of earthquake loads in convex model, and in this study the global energy-bound (GEB) convex model that utilizes the earthquake energy as the main variable was used [8]. The maximum displacement of a SDOF system under earthquake loads is given as follows:

$$S_y(T,\xi) = \frac{\sqrt{E}T^{3/2}}{4\pi\sqrt{2\pi\xi}} \tag{1}$$

where E is the limiting value of the input energy and T and ξ are the natural period and damping ratio, respectively.

If the assumption that the installation of VED does not affect the natural frequency significantly and it only change the damping ratio into ξ_{VED}, the rate of change in response quantities can be simplified. Accordingly, with given response reduction ratio, R, the required damping ratio can be obtained as follows

$$\xi_{VED} = \xi R^{-2} \tag{2}$$

In this study, the VED were designed so that the maximum response of the structure is reduced to half of the responses of the bare structure with $\xi = 1.98\%$ when subjected to the same level of dynamic load. From Eq. (2), the required modal damping ratio is

7.92%, and therefore 5.94% of the modal damping ratio needs to be added to the structure to reduce the response to half.

5. Design of viscoelastic dampers

With the required damping ratio at hand, the modal strain energy method was used to determine the size of VED to provide the structure with the desired amount of modal damping ratio. Modal strain energy method was first developed by Johnson and Kienholz [9], and successfully was applied to evaluate equivalent damping ratio of VED by Chang et al. [7].

The equivalent damping ratio for the ith mode of the structure with VED is expressed as [10].

$$\xi_{iVED} = \frac{E_d^i}{4\pi E^i} = \frac{2\pi\eta_i E^i}{4\pi E^i} = \frac{\eta_i}{2} \tag{3}$$

in which E_d^i and E^i denote, respectively, the energy dissipated in one cycle by the dampers and strain energy of the structure for the ith vibration mode, and η_i denotes an ith modal loss factor, which is expressed as follows

$$\eta_i = \frac{\phi_i^T K_I \phi_i}{\phi_i^T K_R \phi_i} \tag{4}$$

in which ϕ_i is the ith mode of the structure with VED, and K_R and K_I , respectively, denote the real part and imaginary part of complex stiffness matrix of the structure with VED. Thus

$$K_R = K + K_{VED} \tag{5}$$

$$K_I = \eta K_{VED} \tag{6}$$

where K is the stiffness matrix of the bare structure, K_{VED} is the stiffness matrix due to damper contribution alone, η is the loss factor of viscoelastic material. It is noticed that both the K_{VED} and η are functions of modal frequency and operating temperature. Substituting Eq. (4) into Eq. (3), and using Eqs. (5) and (6), we can obtain following formula for equivalent damping ratio

$$\xi_{iVED} = \frac{\eta}{2}\left[1 - \frac{\phi_i^T K \phi_i}{\phi_i^T (K + K_{VED})\phi_i}\right] \tag{7}$$

The size of VED was computed assuming that a pair of VED were installed symmetrically in the first and the second stories where the inter-storey drift computed from numerical analysis was found to be largest. To maximize the relative shear deformation in the dampers, the VED were connected to the structure by Chevron-type supporting braces as shown in Fig. 7.

As the VED and the braces were connected in series, the complex stiffness of the brace-VED system, k^*, is obtained as follows:

$$\frac{1}{k^*} = \frac{1}{k_{VED} + i\eta k_{VED}} + \frac{1}{k_b} \tag{8}$$

where k_{VED} is VED storage stiffness and k_b is brace stiffness. Then, k^* can be expressed as follows:

$$k^* = \frac{\alpha k_{VED} + i\eta\alpha k_{VED}}{\alpha + 1 + i\eta} = \frac{\alpha^2 k_{VED} + (1+\eta)\alpha k_{VED}}{\alpha^2 + 2\alpha + 1 + \eta^2} + i\frac{\eta k_{VED}\alpha^2}{\alpha^2 + 2\alpha + 1 + \eta^2} \tag{9}$$

where α is the stiffness ratio of the brace and the VED, k_b/k_{VED}, and the first and the second term in the right-hand-side of Eq. (9) correspond to the storage stiffness and the loss stiffness, respectively. The property of the VED storage stiffness is shown in Tables 1–3. As α increases, i.e. as the stiffness of brace increases for given VED, the complex stiffness of the combined brace-VED system approaches that of the VED:

$$\lim_{\alpha\to\infty} k^* = k_{VED} + i\eta k_{VED} \tag{10}$$

Fig. 8 plots the variation of the storage stiffness and loss factor as a function of α when $k_{VED} = 1$, $\eta = 0.7$. The storage stiffness of the combined system approaches the stiffness of VED very rapidly, while the loss factor gradually approaches that of VED.

The dimension and stiffness of the supporting steel brace installed in the test model are as follows: length, 5.6 m; cross-sectional dimension (H) 200 ×

Fig. 7. VED with supporting chevron braces.

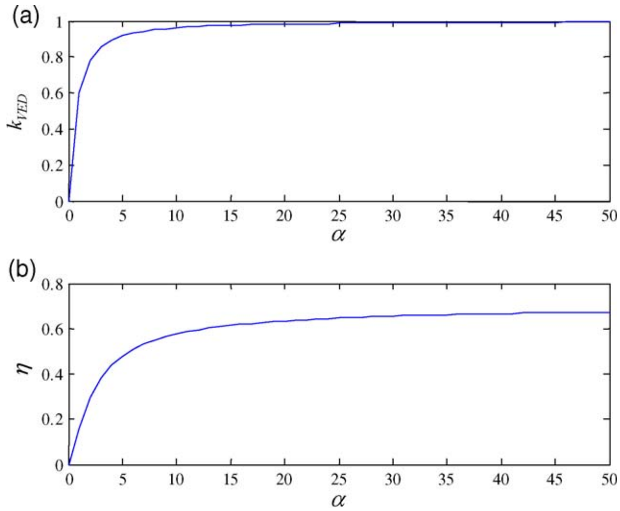

Fig. 8. Storage stiffness and loss factor versus α: (a) storage stiffness, (b) loss factor.

$100 \times 4.5 \times 7$ mm; cross-sectional area, 23.18 cm^2; Young's modulus, 210 GPa, and the slope θ, 57.6°. With this information, the stiffness of a brace member was computed to be 24.9 MN/m. Because two brace members are used in a Chevron-type brace, the stiffness of the Chevron brace is twice that of the single brace, which is 49.8 MN/m.

With the given modal damping ratio required to be supplied by VED, the first step of VED design is to determine VED stiffness to achieve a given target damping ratio. Then the size of VED can be determined from the stiffness. The required stiffness is computed using the modal strain energy method by observing the change in modal damping ratio for various VED stiffness. The stiffness of the brace was not considered in the design process, but was considered later to check the significance of its effect.

Modal properties of the model structure with various size and stiffness of VED were obtained through modal strain energy method. Fig. 9 presents the change in modal damping ratios as a function of the stiffness ratio, k_{VED}/k_i, where k_i is the ith storey stiffness before VED are installed. The equation for the modal strain energy method, Eq. (7), was used for the computation.

An upper bound in the increment of the fundamental damping ratio is shown in the Fig. 9. When VED are placed only in the first storey, the upper bound is 3.3%, while it is 7.58% when they are installed both in the first and the second stories. When the inherent viscous damping ratio of 1.98% is considered, the added damping ratio of 7.58% was found to exceed the total damping ratio required to reduce the response to half, which was computed to be 7.92% by convex model. There-

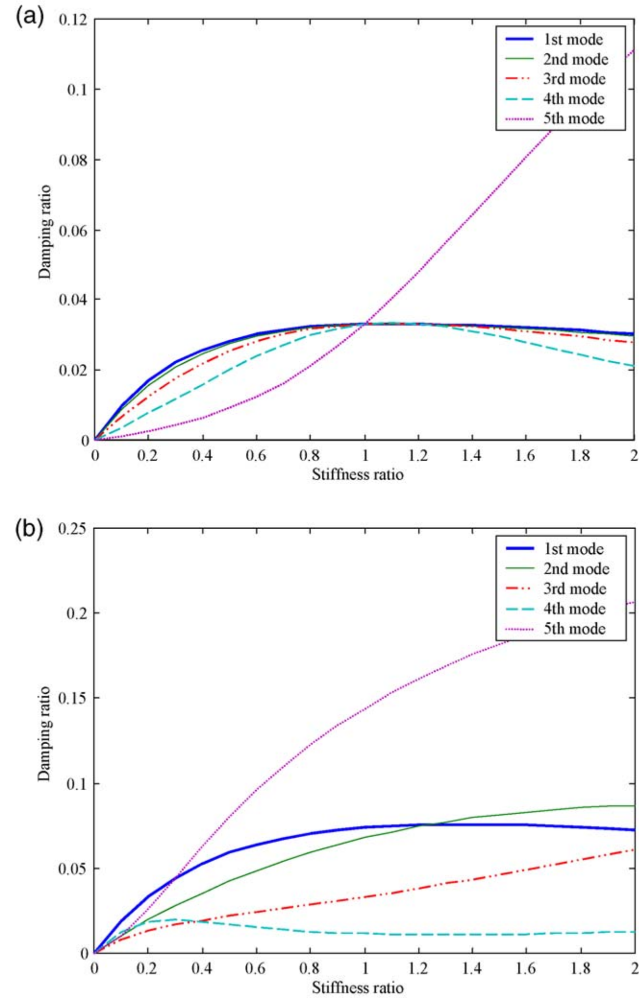

Fig. 9. Change in modal damping ratios as a function of stiffness ratios: (a) VED installed in the first storey; (b) VED installed in the first and second storeys.

fore, if the stiffness of the designed VED, when they are installed both in the first and the second storey, provides the additional damping ratio of 7.58%, then the test structure with VED can be expected to satisfy the given target performance. In this study, the VED were designed so that the maximum stiffness ratio became 1.5 accounting for the error between the theoretical prediction and the test, even though the modal damping ratio reaches the upper bound at the ratio of 1.0. With the storey stiffness $k_i = 2400$ kN/m and two VED installed symmetrically in a storey, the required stiffness of a single VED was computed to be 1830 kN/m.

If two layers of viscoelastic materials are used in a single damper, the required area for a layer is obtained as follows:

$$A = \frac{k_{VED}t}{2G} \tag{11}$$

where G is the shear storage modulus of the viscoelastic material and t is the thickness. The shear storage modulus G is usually a function of frequency and temperature. Since the frequency at which VED operate is approximately the fundamental structural modal frequency of 0.51 Hz, the design excitation frequency was assumed to be 0.5 Hz and G is 0.72 Mpa based on the test results at the temperature of 24 °C, as shown in Fig. 2. In the case of broad-band earthquake excitations, the response of general building such as model structure is governed by fundamental mode, and the design of VED could be performed without much error assuming that VED have properties corresponding to fundamental frequency. The thickness t needs to be decided considering the allowable shear deformation to finalize the area. In this study, the allowable drift of the first storey and the maximum allowable shear strain were set to be 1.2 cm and 0.6, respectively, then the required thickness became $1.2/0.6 = 2$ cm. Finally the area of the layer was computed to be 254 cm^2. Fig. 10 depicts the VED designed in accordance with the above process.

The ratio of brace stiffness to VED stiffness, α, is about 27, and from Eq. (9) the storage and loss stiffness of the brace-VED system become close to those of VED, respectively. Therefore it can be noticed that the storage stiffness of the brace-VED system is almost identical to that of the VED alone and that the loss stiffness decreases slightly from 0.7. The modal damping ratio of the structure with such properties can be computed to be $\xi_1 = 4.98\%$ when VED are installed in the first storey, and $\xi_1 = 8.97\%$ when they are installed in both first and second stories, which is larger than the target value of 7.92% by the convex model.

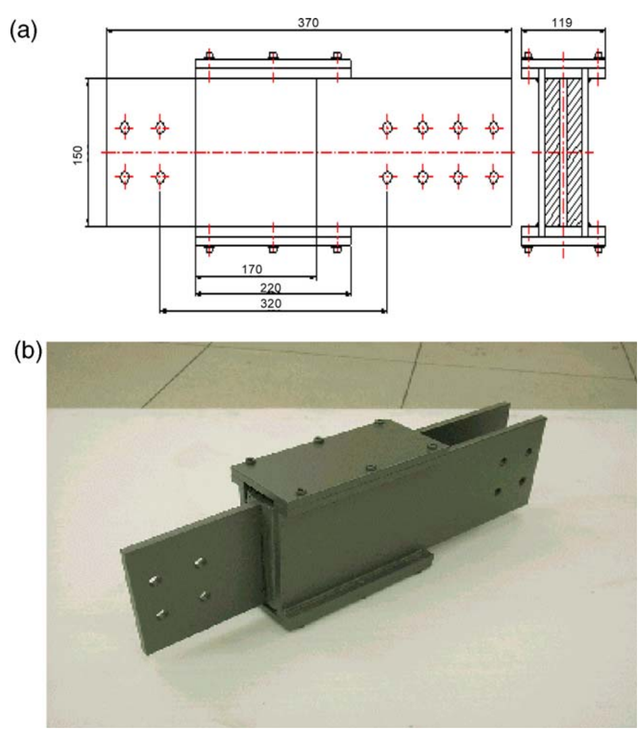

Fig. 10. Viscoelastic dampers used in the experiment: (a) Dimension of the designed VED; (b) Photograph of the VED.

6. Experimental results

Experiments were carried out with three different cases of VED location and temperature: In case 1, VED were installed in the first storey and the experiments were carried out at 30 °C. In case 2, the dampers were installed in the first and the second storey at the same temperature, and in case 3, at 24 °C, VED were located both in the first and the second storey. The structure was vibrated harmonically for 60 s by the HMD with the same forcing frequency scheme as applied in the system identification tests described before.

Fig. 11 plots the transfer functions from sinusoidal HMD force to the fifth floor acceleration of the structure for each case of VED installation. The amplitude of the transfer function decreased significantly with the installation of the VED, and that the amount of the decreased amplitude is larger for the case 3 at lower temperature. The natural frequency, at which the peak occurs, increased as the number of added VED increases. This trend indicates that the stiffness of the structure increases with the installation of VED. The modal damping ratio of the model structure with the added VED, obtained from the half power bandwidth method, were found to be 3.2 and 4.5% for cases 1 and 2, respectively. At the temperature of 24 °C (case 3),

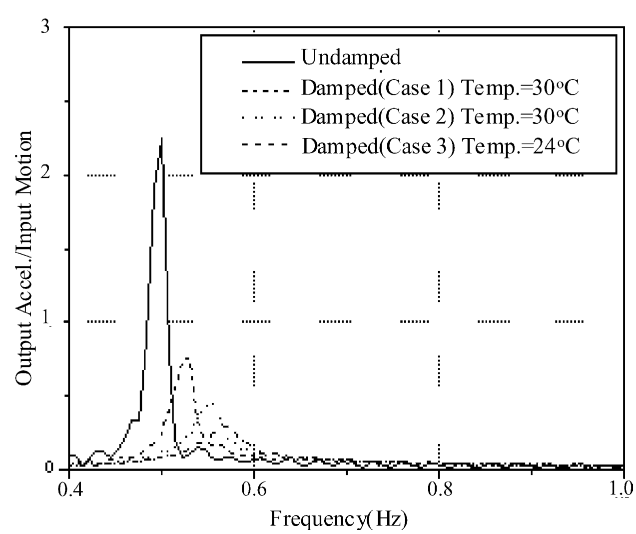

Fig. 11. Transfer functions from sinusoidal HMD acceleration to the fifth floor acceleration.

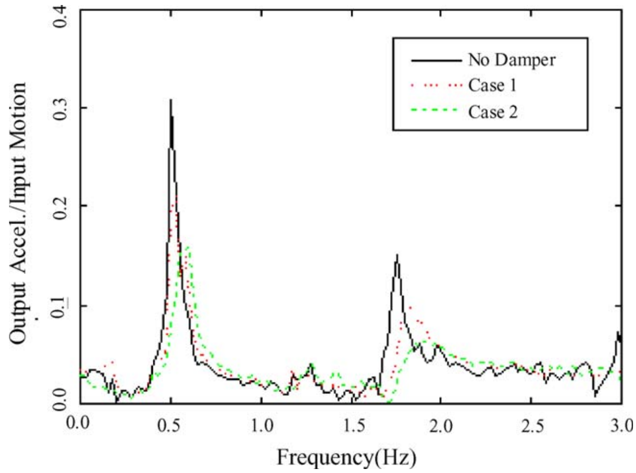

Fig. 12. Transfer function from band-limited white noise acceleration to the fifth floor acceleration.

the modal damping ratio of the structure further increased to 6.6%. These values for the modal damping ratio were much higher than 1.98% of the bare frame, but lower than the target damping ratio 7.84% computed by the modal strain energy method to reduce the structural response to half.

This difference may be attributed from the inaccuracy inherent to the modal strain energy method. Modal strain energy method is based on the assumption that the structure with VED has proportional damping. However, the structure with VED is non-proportional damping system. Accordingly, the violation of this assumption brings the difference between the analytical and experimental results. Another source of error is inaccurate modeling of stiffness matrix. Modal strain energy method indicates that the effect of VED depends on the storey stiffness. Therefore, an accurate knowledge of storey stiffness is essential for accurately estimating the effects of VED. Since, in this study, the stiffness matrix is simply composed to fit the first modal frequency based on the assumptions that accurate mass matrix is given and each storey has identical storey stiffness, the difference may be amplified.

Fig. 12 plots the transfer functions of the structure subjected to band-limited white noise exciting force for cases 1 and 2 of VED installation. Similar to the results for sinusoidal force, the amplitude of the transfer function decreased with the installation of the VED. It also can be observed that as expected the natural frequency, at which the peak occurs, increased as the number of added VED increases. Compared with the results shown in Fig. 11, the effects of VED on response reduction under white noise are not as significant as those under harmonic excitation. This is because harmonic load causes resonant response, and it brings the significantly increased uncontrolled response.

7. Summary

In this study, VED were designed, tested, and installed in a five-storey full-scale model structure to validate their vibration reduction effect. The material properties of the dampers were obtained from cyclic tests and were used in the design process. An excitation scheme using a hybrid mass driver was developed and applied to the model structure. The additional damping ratio required to reduce the maximum response of the structure to a given level was obtained first by convex model. The size of dampers was determined using the modal strain energy method by observing the change in modal damping ratio due to the change in damper stiffness.

Based on the analytical and experimental results, the VED, designed following the procedure mentioned previously, were found to be effective in vibration control of the full-scale structure. From experiments, as expected, the modal damping ratios generally increased as the number of VED increased. In the transfer functions, the acceleration response reduced significantly as a result of VED installation. The response reduction effect of VED was more significant at the temperature of 24 °C than at 30 °C.

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