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The best hydraulic section of horizontal-bottomed parabolic channel section*

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Abstract: The best hydraulic channel section makes the maximum flow capacity for the same flow cross-area, and the minimum cross-area and wetted perimeter for the same discharge. The construction cost can be reduced nearly to the minimum at the same time. The horizontal bottom parabolic section (HBP section) is a composite section. It is important for design to find the best combination form of the horizontal bottom and the parabolic sides. This paper studies the best hydraulic section and its hydraulic characteristics. The explicit formulae are proposed to determine the dimensions and the best combination form of the horizontal bottom and the parabolic sides. These explicit formulae and the parameters make it easy to design the channel. It is shown that the ratios of the surface width to the depth and the bottom width to the depth are constant for the best hydraulic section. The comparisons with the classic parabolic, rectangular, trapezoid, triangular, semi-cubic and horizontal-bottomed semi-cubic sections show that the HBP section has the largest flow capacity and the shortest wetted perimeter for the same flow area, and has the smallest flow area for the same discharge. It is indicated that the parabolic side parts of the best hydraulic HBP section are different from those of the classic section. The results of the best hydraulic section of the classic parabolic channel cannot be applied directly to the HBC section.

Key words: Channel, parabolic shape, horizontal-bottomed, best hydraulic section

Introduction

The best hydraulic cross-section makes the largest flow capacity for a given cross-area. In other words, the cross-area or the wetted perimeter is the minimum for a given conveyance capacity. It can reduce the construction cost nearly to the minimum at the same time[1]. Therefore, the best hydraulic section is always a focus of research. Chow[2] described the explicit formulae of the best hydraulic section for semi-circle, trapezoid, rectangular, parabolic, catenary sections, which helps greatly the designer. Anwar and Clarke[3,4] introduced the freeboard as an additional parameter to be taken into account when designing a power-law channel. Huang et al.[5] proposed a method to design the stable channel using the minimum principle for the river power. It is shown that the shape of the stable channel is circular. Vatankhah[6] presented a type of semi-regular polygon sections such as semi-square, semi-hexagon and semi-octagon sections, and a general solution for the best hydraulic section. Han[7] presented a channel section with horizontal bottom and semi-cubic parabolic shaped sides, deduced the parameters and formula for the best hydraulic section. Wen and Li[8] proposed the hydraulic calculation formula for the horseshoe II type cross-section with flat bottom. Babaeyan-Koopaei et al.[9] proposed a parabolic bottomed triangular shape section using the Lagrange's multiplier method. Abdulrahman[10] considered a composite channel section with a lower trapezoidal section and an upper rectangular section. Froehlich[11] presented the most hydraulically efficient lined channel. Liu et al.[12] presented an optimal hydraulic section of the compound channel with horizontal bottom and vertical sides. Lu et al.[13] studied the

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hydraulic characteristics and the best hydraulic section of the U shaped section. Mohanty and Khatua[14] studied the discharge and its distribution in compound channels. Han et al.[15] deduced the formulae of the best hydraulic section for the ice-covered trapezoidal channel. Maleki and Khan[16] evaluated the effect of different channel shapes for accuracy and efficiency of the open channel flow.

There are varieties of forms of the channel section. The parabolic section is one of them. It was suggested[17,2,7] that the unlined canals and the irrigation furrows all might be approximated by a stable parabolic shape. Therefore, the channels can be made more hydraulically stable by initially constructing them in a parabolic shape. It was shown[18,19] that the discharge capacity of a parabolic section is larger than that of a trapezoid section under the same conditions. The cost of the former is less than the latter[20].

The horizontal bottomed parabolic section is composed of a horizontal bottom and two parabolic sides. There are three issues to be considered. The first one concerns the best combination form of the bottom and the sides. The second one concerns the flow capacity under the best hydraulic condition as compared with the classic parabolic section. The third one concerns the possibility of direct applications of the best hydraulic section of the classic parabolic section to the horizontal bottomed parabolic section.

In this paper, the best hydraulic section and the explicit equations are derived to be used to directly calculate the dimensions of the horizontal-bottomed parabolic section. The comparisons with the classic parabolic section and other types of sections are presented.

1. The classic parabolic section

The shape of the classic parabolic section (as shown in Fig.1) is defined by

$$y = ax^2 \tag{1}$$

in which a is the shape parameter, x is the horizontal coordinate, y is the vertical coordinate.

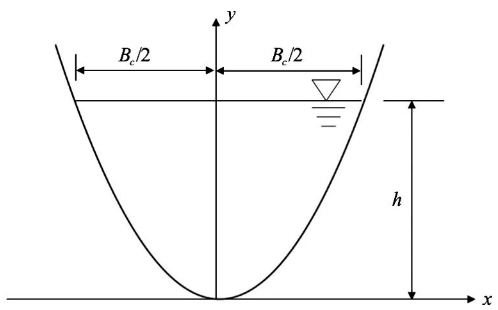

Fig.1 Classic parabolic section

From Fig.1, it follows that, if $x = B_c/2$, then $y = h$. Therefore, a can be computed by

$$a = 4\frac{h}{B_c^2} \tag{2}$$

where h, B_c are the water depth and the water surface width of the classic parabolic section.

The flow area and the wetted perimeter can be deduced using the integration method[19,21], as

$$A_c = 2\left(h\frac{B_c}{2} - \int_0^{B_c/2} y\mathrm{d}x\right) = hB_c - \frac{1}{12}aB_c^3 = \frac{2}{3}B_c h \tag{3}$$

$$P_c = \int_0^{B_c/2} \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x =$$

$$\frac{1}{2}\frac{B_c\sqrt{a^2B_c^2+1}\,a + \ln(aB_c + \sqrt{a^2B_c^2+1})}{a} =$$

$$\frac{1}{8}\frac{B_c^2}{h}\left[4\frac{h}{B_c}\sqrt{16\frac{h^2}{B_c^2}+1} + \ln\left(4\frac{h}{B_c} + \sqrt{16\frac{h^2}{B_c^2}+1}\right)\right] \tag{4}$$

where A_c is the flow area of the classic parabolic section, P_c is the wetted perimeter of the classic parabolic section.

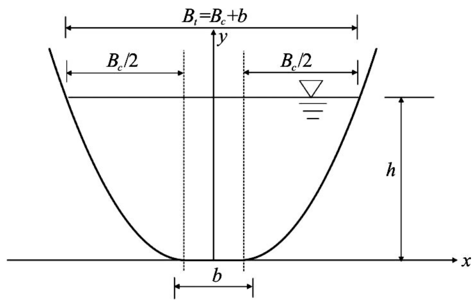

Fig.2 Horizontal-bottomed parabolic section

2. Characteristics of horizontal-bottomed parabolic section

The horizontal-bottomed parabolic section (as shown in Fig.2) can be expressed by

$$y = a_t\left(x+\frac{b}{2}\right)^2, \quad x \le -\frac{b}{2} \tag{5a}$$

$$y=0\,,\ -\frac{b}{2}<x<\frac{b}{2} \tag{5b}$$

$$y=a_t\left(x-\frac{b}{2}\right)^2,\ x\geq\frac{b}{2} \tag{5c}$$

where a_t is the shape parameter of the horizontal-bottomed parabolic section, b is the bottom width.

Obviously, the flow area and the wetted perimeter of the horizontal-bottomed parabolic section can be obtained from the results of the classic parabolic section.

$$B_t=B_c+b \tag{6}$$

$$A_t=A_c+bh=\frac{2}{3}B_c h+bh \tag{7}$$

$$P_t=P_c+b=\frac{1}{8}\frac{B_c^{\,2}}{h}\left[4\frac{h}{B_c}\sqrt{16\frac{h^2}{B_c^{\,2}}+1}+\ln\left(4\frac{h}{B_c}+\sqrt{16\frac{h^2}{B_c^{\,2}}+1}\right)\right]+b \tag{8}$$

where A_t is the flow area of the horizontal-bottomed parabolic section, P_t is the wetted perimeter of the horizontal-bottomed parabolic section.

3. Optimum hydraulic section of horizontal-bottomed parabolic section

3.1 *Characteristics of the best hydraulic section*

The discharge of the uniform flow is expressed using the Manning's formula as[2]

$$Q_t=\frac{1}{n}\frac{A_t^{5/3}i^{1/2}}{P_t^{2/3}} \tag{9}$$

where Q_t is the flow discharge for the parabolic section with horizontal bottom, n is the roughness, P_t is the wetted perimeter, i is the channel bottomed horizontal slope.

With dimensionless variables, $\eta_t=B_c/h$, $\beta_t=b/h$, A_t in Eq.(7) and P_t in Eq.(8) become

$$A_t=\beta_t h^2+\frac{2}{3}\eta_t h^2 \tag{10}$$

$$P_t=\frac{1}{8}\left[4\frac{1}{\eta_t}\sqrt{16\eta_t^{-2}+1}+\ln(4\eta_t^{-1}+\sqrt{16\eta_t^{-2}+1})\right]h\eta_t^2+\beta_t h \tag{11}$$

Equation (10), Eq.(11) and Eq.(9) indicate that A_t, P_t and Q_t can be determined by η_t, β_t and h.

According to the definition of the channel optimum hydraulic section, the best hydraulic section is the type of shape with the maximum discharge for a given area or with the minimum flow area for a given discharge. So the optimization model can be defined by

$$\text{Minimize}\quad A_t=A_t(\eta_t,\beta_t,h) \tag{12a}$$

$$\text{Subject to}\quad \Phi(\eta_t,\beta_t,h)=Q_t-\frac{1}{n}\frac{A_t^{5/3}i^{1/2}}{P_t^{2/3}}=0 \tag{12b}$$

Based on the Lagrange's multiplier optimization method, the optimal problem can be described as

$$\frac{\partial A_t}{\partial\eta_t}+\lambda\frac{\partial\Phi}{\partial\eta_t}=0 \tag{13a}$$

$$\frac{\partial A_t}{\partial\beta_t}+\lambda\frac{\partial\Phi}{\partial\beta_t}=0 \tag{13b}$$

$$\frac{\partial A_t}{\partial h}+\lambda\frac{\partial\Phi}{\partial h}=0 \tag{13c}$$

where λ is the Lagrange's multiplier.

Eliminating λ from Eq.(13a) and Eq.(13b) results in

$$\frac{\partial A_t}{\partial\eta_t}\frac{\partial\Phi}{\partial\beta_t}=\frac{\partial\Phi}{\partial\eta_t}\frac{\partial A_t}{\partial\beta_t} \tag{14a}$$

Eliminating λ from Eq.(13a) and Eq.(13c) results in

$$\frac{\partial A_t}{\partial\eta_t}\frac{\partial\Phi}{\partial h}=\frac{\partial A_t}{\partial h}\frac{\partial\Phi}{\partial\eta_t} \tag{14b}$$

From Eq.(12b), the first partial derivative of Φ with respect to the variables η_t, β_t and h are expressed as

$$\frac{\partial}{\partial\eta_t}\Phi(\eta_t,\beta_t,h)=-\frac{5}{3}\frac{A_t^{2/3}i^{0.5}\dfrac{\partial A_t}{\partial\eta_t}}{nP_t^{2/3}}+\frac{2}{3}\frac{A_t^{5/3}i^{0.5}\dfrac{\partial P_t}{\partial\eta_t}}{nP_t^{5/3}} \tag{15a}$$

$$\frac{\partial}{\partial \beta}\Phi(\eta_t,\beta_t,h)=-\frac{5}{3}\frac{A_t^{2/3}i^{0.5}\frac{\partial A_t}{\partial \beta_t}}{nP_t^{2/3}}+\frac{2}{3}\frac{A_t^{5/3}i^{0.5}\frac{\partial P_t}{\partial \beta_t}}{nP_t^{5/3}} \tag{15b}$$

$$\frac{\partial}{\partial h}\Phi(\eta_t,\beta_t,h)=-\frac{5}{3}\frac{A_t^{2/3}i^{0.5}\frac{\partial A_t}{\partial h}}{nP_t^{2/3}}+\frac{2}{3}\frac{A_t^{5/3}i^{0.5}\frac{\partial P_t}{\partial h}}{nP_t^{5/3}} \tag{15c}$$

Substituting Eq.(15a) and Eq.(15b) into Eq.(14a) and simplifying, we have

$$-\frac{2}{3}\frac{1}{nP_t^{5/3}}A_t^{5/3}i^{0.5}\left(\frac{\partial A_t}{\partial \eta_t}\frac{\partial P_t}{\partial \beta_t}-\frac{\partial A_t}{\partial \beta_t}\frac{\partial P_t}{\partial \eta_t}\right)=0 \tag{16a}$$

In the same way, substituting Eq.(15a) and Eq.(15c) into Eq.(14b) results in

$$-\frac{2}{3}\frac{1}{nP_t^{5/3}}A_t^{5/3}i^{0.5}\left(\frac{\partial A_t}{\partial \eta_t}\frac{\partial P_t}{\partial h}-\frac{\partial A_t}{\partial h}\frac{\partial P_t}{\partial \eta_t}\right)=0 \tag{16b}$$

Therefore, the conditions for the optimal hydraulic section of the horizontal-bottomed parabolic section become

$$\frac{\partial A_t}{\partial \eta_t}\frac{\partial P_t}{\partial \beta_t}=\frac{\partial A_t}{\partial \beta_t}\frac{\partial P_t}{\partial \eta_t},\quad \frac{\partial A_t}{\partial \eta_t}\frac{\partial P_t}{\partial h}=\frac{\partial A_t}{\partial h}\frac{\partial P_t}{\partial \eta_t} \tag{16c}$$

From Eq.(10) and Eq.(11), the partial derivatives of A_t and P_t with respect to the variables η_t, β_t and h are:

$$\frac{\partial A_t}{\partial \eta_t}=\frac{2}{3}h^2,\quad \frac{\partial A_t}{\partial \beta_t}=h^2,\quad \frac{\partial A_t}{\partial h}=2\beta_t h+\frac{4}{3}\eta_t h \tag{17a}$$

$$\frac{\partial P_t}{\partial \eta_t}=\frac{1}{4}\eta_t[\ln(4+\sqrt{\eta_t^2+16})-\ln(\eta_t)]h \tag{17b}$$

$$\frac{\partial P_t}{\partial \beta_t}=h,$$

$$\frac{\partial P_t}{\partial h}=\frac{1}{8}\ln(4+\sqrt{\eta_t^2+16})\eta_t^2-\frac{1}{8}\ln(\eta_t)\eta_t^2+$$

$$\frac{1}{2}\sqrt{\eta_t^2+16}+\beta_t \tag{17c}$$

Substituting Eq.(17) into Eq.(16c), after simplifying, the following expression is obtained:

$$\frac{\eta_t}{4}[(4+\sqrt{\eta_t^2+16})-\ln(\eta_t)]=\frac{2}{3} \tag{18a}$$

$$2\eta_t[\ln(4+\sqrt{\eta_t^2+16})-\ln(\eta_t)](3\beta_t+2\eta_t)=$$

$$\ln(4+\sqrt{\eta_t^2+16})\eta_t^2+4\sqrt{\eta_t^2+16}+8\beta_t \tag{18b}$$

Using the bisection root finding method or the Newton iterative method to solve the first equation of Eqs.(18), the parameter η_t of the best hydraulic section of the horizontal-bottomed parabolic section can be obtained as

$$\eta_t=\frac{B_c}{h}=1.6439 \tag{19}$$

Substituting η_t into the second equation of Eqs.(18), the parameter β is obtained as

$$\beta_t=\frac{b}{h}=0.5184 \tag{20}$$

From Eq.(19) and Eq.(20), one obtains

$$\frac{B_t}{h}=\frac{B_c+b}{h}=2.1623 \tag{21}$$

Equation (19), Eq.(20) and Eq.(21) describe the best combination form using the horizontal bottom and parabolic shape sides.

3.2 *Explicit formulae of the best hydraulic section for design*

(1) Computing discharge Q_t when h is known

In design, it is often required to compute the discharge when the section dimensions are known. Now let $\eta_t=B_c/h$, $\beta_t=b/h$ in Eq.(2), Eq.(10) and Eq.(11), the explicit formulae of the optimum hydraulic section for a_t, A_t, P_t and Q_t can be deduced as

$$a_t=1.4801h^{-1}\quad\text{or}\quad a_t h=1.4801 \tag{22}$$

$$A_t=1.6143h^2 \tag{23}$$

$$P_t=3.2287h \tag{24}$$

$$Q_t=1.0170\frac{h^{8/3}\sqrt{i}}{n} \tag{25}$$

(2) Computing h, b, B_t, A_t, P_t, when Q is known

Sometimes, it is necessary to calculate the dimensions such as h, b, B_c, B_t for a known flow discharge. From Eq.(25), the normal water depth h for the best hydraulic section can be obtained firstly

$$h=0.9937\left(\frac{Q_t n}{\sqrt{i}}\right)^{3/8} \tag{26a}$$

Then using the relationships: $b=\beta_t h$, $B_c=\eta_t h$ and $B_t=B_c+b$, the values of b and B_t can be obtained. $b=0.5184h$, $B_c=1.6439h$, $B_t=2.1623h$.

Substituting Eq.(26a) into Eq.(6), Eq.(7) and Eq.(8), the following formulae for B_t, A_t and P_t are obtained as

$$B_t=2.1487\left(\frac{Q_t n}{\sqrt{i}}\right)^{3/8} \tag{26b}$$

$$A_t=1.5941\left(\frac{Q_t n}{\sqrt{i}}\right)^{3/4} \tag{26c}$$

$$P_t=3.2084\left(\frac{Q_t n}{\sqrt{i}}\right)^{3/8} \tag{26d}$$

These parameters (Eq.(19) to Eq.(22)) and the explicit equations(Eq.(23) to Eq.(26d)) are easy to use for the design of the best hydraulic section.

3.3 *The normal depth and the critical depth of the best hydraulic section*

Obviously, Eq.(26a) is the explicit formula for the normal depth of the best hydraulic section.

The general critical equation is expressed as[2]

$$\frac{\alpha Q^2}{g}=\frac{A^3}{B} \tag{26e}$$

where g is the gravitational acceleration, α is the energy correction factor.

Substituting Eqs.(7) and (21) into Eq.(26e), the critical depth h_k is obtained as

$$h_k=0.8287\frac{\sqrt[5]{Q^2 g^4}}{g} \tag{26f}$$

3.4 *Verification by using numerical optimization method*

For verifying the results of the best hydraulic section from another angle, the optimization variables are set as a, b, c, according to the definition of the best hydraulic section and Eq.(2), Eq.(7) and Eq.(8), The numerical optimization model can be expressed as

$$\text{Minimize}\quad A_t=A_t(B_c,b,h)=\frac{2}{3}B_c h+bh \tag{27a}$$

$$\text{Subject to}\quad \Phi(B_c,b,h)=Q_t-\frac{1}{n}\frac{A_t^{5/3}i^{1/2}}{P_t^{2/3}}=0 \tag{27b}$$

where P_t can be computed by Eq.(8).

Here, the optimization algorithm of the sequential quadratic programming (SQP) is adopted to verify the above analytical solution. The initial values of B_c, b, h are limited to any values in the range of $[0.0001, 10\,000]$. It is shown that for any initial values of B_c, b, h, the final optimization result is the same, as Eq.(19) and Eq.(20).

3.5 *Examples*

Example 1: A channel of the designed water depth of 2.2 m, the bed roughness of 0.014, and the bottom slope of 1/12 000. Now we make this channel to have the largest flow capacity for the same flow area.

From Eq.(20) and Eq.(21), one can obtain the bottom width $b=1.1405\,\text{m}$, and the total water surface $B_t=4.7571\,\text{m}$. From Eq.(22), the shape factor is obtained $a=0.6728$. From Eq.(23) and Eq.(24), one can obtain the flow area $a=7.8134\,\text{m}^2$, the wetted perimeter $p_t=7.1031\,\text{m}$.

Example 2: A channel of the flow discharge of 9.0 m^3/s, the bed roughness of 0.014, and the bottom slope of 1/15 000. Now we design a channel having the largest flow capacity for the same flow area.

From Eq.(26a) and Eqs.(19) through (22), one obtains the normal water depth $h=2.7728\,\text{m}$, then $b=\beta_t h=1.4374\,\text{m}$, $B_c=\eta_t h=4.5583\,\text{m}$, $B_t=B_c+b=5.9956\,\text{m}$, $a=1.4801h^{-1}=0.5338$.

Secondly, with all known values in Eq.(26c) and Eq.(26d), the flow area and the wetted perimeter are obtained as

$$A_t=12.4116\,\text{m}^2,\quad p_t=8.9524\,\text{m}$$

We have solved these two examples also by using the nonlinear optimization method. The results are the same as above. However, it is much easier using these analytical solutions, the explicit parameters and the formulae to design the channels than using the nonlinear optimization method.

4. Comparison with classic parabolic shaped section

4.1 *Best hydraulic section of classic parabolic shaped section*

In Fig.2, with the dimensionless variable $\eta_c=B_c/h$, A_c (Eq.(3)) and P_c (Eq.(4)) of the classic parabolic section become

$$A_c = \frac{2}{3}\eta_c h^2 \tag{28a}$$

$$P_c = \frac{1}{8}\left[4\frac{1}{\eta_c}\sqrt{16\eta_c^{-2}+1} + \ln(4\eta_c^{-1} + \sqrt{16\eta_c^{-2}+1})\right]h\eta_c^2 \tag{28b}$$

From Eq.(28a) and Eq.(28b), the area and the wetted perimeter can be determined by η_c and h in the classic parabolic section. In the same way, using the Lagrange's multiplier method, the condition for the best hydraulic section is

$$\frac{\partial A_c}{\partial \eta_c}\frac{\partial P_c}{\partial h} = \frac{\partial P_c}{\partial \eta_c}\frac{\partial A_c}{\partial h} \tag{29}$$

From Eq.(27) and Eq.(28), the partial derivatives of A_c and P_c with respect to the variables η_c, h are

$$\frac{\partial A_c}{\partial \eta_c} = \frac{2}{3}h^2, \quad \frac{\partial A_c}{\partial h} = \frac{4}{3}\eta_c h \tag{30a}$$

$$\frac{\partial P_c}{\partial \eta_c} = -\frac{1}{4}\eta_c[\ln(\eta_c) - \ln(4+\sqrt{\eta_c^2+16})]h \tag{30b}$$

$$\frac{\partial P_c}{\partial h} = \frac{1}{8}\left[4\frac{1}{\eta_c}\sqrt{16\eta_c^{-2}+1} + \ln(4\eta_c^{-1} + \sqrt{16\eta_c^{-2}+1})\right]\eta_c^2 \tag{30c}$$

Substituting Eq.(30) into Eq.(29), the following equation is obtained

$$\frac{1}{12}[-\ln(\eta_c)\eta_c^2 + \ln(4+\sqrt{\eta_c^2+16})\eta_c^2 + 4\sqrt{\eta_c^2+16}]h^2 = -\frac{1}{3}\eta_c^2[\ln(\eta_c) - \ln(4+\sqrt{\eta_c^2+16})]h^2 \tag{31}$$

Solving Eq.(31) , the value of η_c of the best hydraulic section for the classic parabolic section is obtained

$$\eta_c = \frac{B_c}{h} = 2.0555 \quad \text{or} \quad \frac{1}{\eta_c} = \frac{h}{B_c} = 0.4865 \tag{32}$$

Here η_c is the parameter of the best hydraulic section for the classic parabolic section.

Substituting Eq.(32) into Eq.(2), Eq.(27) and Eq.(28), the explicit formulae for a_c, A_c and P_c are obtained

$$a_c = 0.9467h^{-1} \tag{33}$$

$$A_c = 1.3703h^2 \tag{34}$$

$$P_c = 2.9982h \tag{35}$$

Substituting Eq.(34) and Eq.(35) into the uniform flow formula $Q_c = n^{-1}A_c^{5/3}i^{1/2}/P_c^{2/3}$, the relationship between Q_c and h for the classic parabolic best hydraulic section is obtained as

$$Q_c = 0.8131\frac{h^{8/3}\sqrt{i}}{n} \tag{36}$$

The normal water depth under the best hydraulic section conditions can be computed by

$$h = 1.0807\left(\frac{Q_c n}{\sqrt{i}}\right)^{3/8} \tag{37a}$$

Substituting Eq.(37a) into Eq.(32), Eq.(34) and Eq.(35), the explicit equations for design are obtained for a given discharge

$$B_c = 2.2213\left(\frac{Q_c n}{\sqrt{i}}\right)^{3/8}, \quad A_c = 1.6004\left(\frac{Q_c n}{\sqrt{i}}\right)^{3/4},$$

$$P_c = 3.2401\left(\frac{Q_c n}{\sqrt{i}}\right)^{3/8} \tag{37b}$$

4.2 *Best hydraulic section of trapezoid section*

It is known that the ratio of the bottom width to the depth is $b/h = 2(\sqrt{m^2+1} - m)$ for the best hydraulic section of the trapezoid section. When $m = \sqrt{3}/3$, the flow discharge is the maximum[2]. Using the same method, the characteristics of the best hydraulic section for the trapezoid section can be obtained (as listed in Table 1).

4.3 *Comparison with classic parabolic section and trapezoid section*

Assuming $\varepsilon = Qn/\sqrt{i}$, the properties of the best hydraulic section for the classic parabolic and HBC sections are listed in Table 1.

From Eqs.(19) through (26d) and the comparisons listed in Table 1, it follows that

Table 1 Properties of the best hydraulic section for trapezoid, classic parabolic and HBC sections

Section type	B_c / h	Shape factor a	Normal water depth/m	Water surface width/m	Cross area/m^2	Wetted perimeter/m
Trapezoid section	$b/h = 1.1547$	$m = \sqrt{3}/3$	$h = 0.9678\varepsilon^{3/8}$	$2.2351\varepsilon^{3/8}$	$1.6224\varepsilon^{3/8}$	$3.3527\varepsilon^{3/8}$
Classic parabolic section	2.0555	$0.9467/h$	$h = 1.0807\varepsilon^{3/8}$	$2.2213\varepsilon^{3/8}$	$1.6004\varepsilon^{3/8}$	$3.2401\varepsilon^{3/8}$
HBC section	1.6439	$1.4801/h$	$h = 0.9937\varepsilon^{3/8}$	$2.1487\varepsilon^{3/8}$	$1.5941\varepsilon^{3/8}$	$3.2084\varepsilon^{3/8}$

Table 2 The best hydraulic section of trapezoid, HBP and HBC sections for Example 1

Section type	a	Bottom width/m	B_c / m	Water surface width/m	Cross area/m^2	Wetted perimeter/m	Discharge
Classic parabolic section	0.6312	0	3.0832	3.0832	3.0832	4.4972	1.3981
HBC section	0.9867	0.7776	2.4659	3.2435	3.6323	4.8430	1.7487

(1) The water depth flow area, the wetted perimeter and the water surface width of the horizontal-bottomed parabolic section are all smaller than those of the classic parabolic and trapezoid sections for a given flow discharge for the best hydraulic section.

(2) It is indicated that for a given area or wetted perimeter, the flow discharge of the HBC section is larger than that of the classic parabolic and trapezoid sections.

(3) The shape factors a and B_c / h of the HBC and classic parabolic sections are different. Therefore, the side shape of the best hydraulic section of the classic parabolic section cannot be applied directly to the horizontal bottomed parabolic section.

4.4 *Applications*

Example 1: A canal with parabolic sides and horizontal bottom with the bed roughness $n = 0.014$, the bottom slope $i = 1/15\,000$ and the water depth $h =$ 1.5 m. Now it is necessary to design the best hydraulic section, and calculate A_t, P_t, Q_t, then compare the results with those of the classic parabolic section.

(1) The results of the section with horizontal bottom and parabolic sides

$$a = 1.4801h^{-1} = 0.9867\,,\ \ B_c = \eta_t h = 2.4659\ \text{m}\,,$$

$$b = \beta_t h = 0.7776\ \text{m}\,,\ \ B_t = B_c + b = 3.2435\ \text{m}$$

Then substituting all known values into Eqs.(22) through (25), we obtain

$$A_1 = 1.6143h^2 = 3.6323\ \text{m}^2\,,$$

$$p_t = 3.2287h = 4.8430\ \text{m}\,,$$

$$Q_t = 1.0170h^{8/3}\sqrt{i}\,n^{-1} = 1.7487\ \text{m}^3/\text{s}$$

(2) The results of classic parabolic section

Substituting all known values into Eq.(32) and Eq.(33), we have

$$a = 0.9467h^{-1} = 0.6312\,,\ \ B_c = \eta_c h = 3.0832\ \text{m}$$

Similarly, substituting all known values into Eqs.(34) through (36), we obtain

$$A_c = 3.0832\ \text{m}^2\,,\ \ p_c = 4.4972\ \text{m}\,,\ \ Q_c = 1.3981\ \text{m}^3/\text{s}$$

The results are listed in Table 2.

Example 2: A channel with parabolic sides and horizontal bottom with $n = 0.012$, $i = 1/20\,000$ and $Q = 3.2\ \text{m}^3/\text{s}$. Now design a channel using the best hydraulic section method.

(1) The results of using horizontal bottom parabolic section

Firstly, substituting the flow discharge and other known values into Eq.(26a) and Eqs.(19) through (22). The water depth h and other dimensions can be calculated:

$$h = 0.9937\left(\frac{Qn}{\sqrt{i}}\right)^{3/8} = 1.8742\ \text{m}\,,\ \ b = \beta_t h = 0.9716\ \text{m}\,,$$

$$B_c = \eta_t h = 3.0811\ \text{m}\,,\ \ B_t = B_c + b = 4.0527\ \text{m}\,,$$

$$a = 1.4801h^{-1} = 0.7897$$

Secondly, substituting all known values into Eq.(26c) and Eq.(26d), we obtain

$$A_t = 5.6709\ \text{m}^2\,,\ \ P_t = 6.0513\ \text{m}$$

Table 3 The best hydraulic section of trapezoid, HBP and HBC sections for Example 2

Section type	Bottom width/m	Water depth/m	Water surface width/m	Cross area/m^2	Wetted perimeter/m	Remarks
Trapezoid section	2.1078	1.8254	4.2156	5.7715	6.3235	$m=\sqrt{3}/3$
Classic parabolic section	0	2.0383	4.1897	5.6932	6.1111	$a=0.4645$
HBC section	0.9716	1.8742	4.0527	5.6709	6.0514	$a=0.7897$

Table 4 Properties of best hydraulic section for different types of sections ($\varepsilon=Qn/\sqrt{i}$)

Section type	Water surface width/m	Cross area/m^2	Wetted perimeter/m
Classic parabolic section	$2.2213\varepsilon^{3/8}$	$1.6004\varepsilon^{3/8}$	$3.2401\varepsilon^{3/8}$
Horizontal-bottomed parabolic section	$2.1487\varepsilon^{3/8}$	$1.5941\varepsilon^{3/8}$	$3.2084\varepsilon^{3/8}$
Rectangular section	$1.8340\varepsilon^{3/8}$	$1.6818\varepsilon^{3/8}$	$3.6680\varepsilon^{3/8}$
Triangular section	$2.5940\varepsilon^{3/8}$	$1.6818\varepsilon^{3/8}$	$3.6680\varepsilon^{3/8}$
Trapezoid section	$2.2351\varepsilon^{3/8}$	$1.6224\varepsilon^{3/8}$	$3.3527\varepsilon^{3/8}$
Semi-cubic	$2.3355\varepsilon^{3/8}$	$1.6213\varepsilon^{3/8}$	$3.3468\varepsilon^{3/8}$
Horizontal-bottomed semi-cubic	$2.1845\varepsilon^{3/8}$	$1.6015\varepsilon^{3/8}$	$3.2456\varepsilon^{3/8}$

(2) The results of using classic parabolic section

Substituting all known values into Eq.(37a), Eq.(32) and Eq.(33), we have

$$h=1.0807\left(\frac{Qn}{\sqrt{i}}\right)^{3/8}=2.0384\text{ m},\ B_c=\eta h=4.1897\text{ m},$$

$$a=0.9467h^{-1}=0.4645$$

Secondly, substituting all known values into Eq.(37b), we obtain $A_t=5.6932\text{ m}^2$, $P_t=6.1111\text{ m}$.

The results are listed in Table 3.

The results from two examples show that these explicit formulae are easy to use in the design of the best hydraulic section. The comparisons in the Example 2 show that the flow area and the wetted perimeter of the HBC section are smaller than those of the classic parabolic section.

5. Comparison with other types of sections

In the same way, one can study the characteristics of the rectangular, trapezoid, triangular, semi-cubic and horizontal-bottomed semi-cubic sections under the best hydraulic conditions. The comparisons between the horizontal-bottomed parabolic section and other nine types of sections are listed in Table 4.

$$B_t=2.148715\left(\frac{Q_t n}{\sqrt{i}}\right)^{3/8} \tag{38}$$

$$A_t=1.594094\left(\frac{Q_t n}{\sqrt{i}}\right)^{3/4} \tag{39}$$

$$P_t=3.208374\left(\frac{Q_t n}{\sqrt{i}}\right)^{3/8} \tag{40}$$

The comparisons show that the flow area and the wetted perimeter of the HBC section are the smallest among all seven types of sections listed in Table 4 for the same discharge. In other words, the discharge is the largest for a given flow area. The construction cost of the channel is mainly related with the earthwork excavation, lining and land requisition expenses. In general, the land requisition accounts for a small proportion. Therefore, the HBC section is an economic section.

6. Conclusions

To find the best combination form of the horizontal bottom and parabolic sides is important for the design of the horizontal bottom parabolic channel section. In this paper, the section parameters and the explicit formulae of the best hydraulic section are derived and can be used to calculate the channel dimensions directly. It is shown that the ratio of the horizontal bottom to the water depth b/h is 0.5184, the ratio of the total water surface width to the water depth B_t/h is 2.1623, the product of the shape factor and the water depth $a_t h$ is 1.4801. The comparisons with other nine types of sections (the classic parabolic, rectangular, trapezoid, triangular, semi-cubic and horizontal-bottomed semi-cubic sections) show that the HBP section has the largest flow capacity and the shortest wetted perimeter for the same flow area. On the other hand, it has the smallest flow area for the same discharge. Comparing the shape parameters (a, a_t and B_c), it is shown that the parameters a and B_c / h of the classic parabolic and HBC sections are different. Therefore, the side shape of the best hydraulic section of the classic parabolic section cannot be applied directly to the horizontal bottomed parabolic section.

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