Evaluation of Structural Reliability for Reinforced Concrete Buildings Exposed to Corrosion

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ABSTRACT: A probabilistic criterion proposed by Cornell et al (2002) and extended by Tolentino et al (2012) is used here to study the effect of long-term material degradation due to corrosion of reinforced concrete buildings located in the Pacific Coast of Mexico. The criterion considers, by means of closed form mathematical expressions, the simultaneous variation of the structural capacity and of the structural demand, over time. The structural reliability is represented in terms of two alternative indicators: a) the expected number of failures over a time interval, and b) the confidence factors as functions of time, within a Demand and Capacity Factor Design format. The reliability indicators are extended here in order to take into account the structural deterioration due to corrosion over a time interval. Both, aleatory and epistemic uncertainties are taken into account. The structural reliability is evaluated for a 4-story building subjected to a set of real seismic ground motions recorded in Acapulco Bay, Mexico.

1. INTRODUCTION

Reinforced concrete is one of the most common materials in construction; however, it is susceptible to present unacceptable reliability levels when it is exposed, after an interval of time, to the attack of aggressive agents in coastal areas, to icing or snow, or in sites with high levels of contamination. The factors that affect the corrosion of reinforcing steel are several, including temperature, humidity, ocean acidification, airborne pollutants, etc. Depending on the exposure conditions, each of them can influence the time of initiation and/or progression of corrosion.

The cause of failure of reinforced concrete structures subjected to corrosion is due to the loss of structural capacity resulting from the properties deterioration, which affects the adherence to the concrete, section cracks or even spalling (Andrade et al. (1993); Youping and Weyers (1998); Castaneda et al. (1997)).

The annual cost of corrosion worldwide is estimated to exceed $US 1.8 trillion, which translates to 3% to 4% of the Gross Domestic Product (GDP) of industrialized countries (Schmitt, 2009). A measure to reduce the social and economic problems that it involves can be solved by providing reliability-based tools for inspection and maintenance planning of the infrastructure and structures affected by corrosion. The planning will depend, in part, on the estimation of the structural reliability considering that both structural capacity and structural demand varies over time due to the deterioration of the structural members caused by corrosion.

2. CORROSION OF REINFORCED CONCRETE

Corrosion induced by chloride penetration occurs mainly in structures exposed to marine environments. The chloride ions are present in seawater; however, as the wind shifts as breeze, these are deposited in structures close to the seafront.

In sections 2.1 and 2.2 several basic definitions related with the corrosion process are reviewed.

2.1. Time when corrosion begins

The penetration of chloride ions in concrete is difficult to model. Commonly, the law of diffusion (Fick's law) is used. If it is assumed that the concentration of chlorides in the concrete surface and the diffusion coefficient D_c for concrete are independent, then it is possible to establish the following:

$$\frac{\partial C(x,t)}{\partial t} = D_c \frac{\partial^2 C(x,t)}{\partial x^2} \qquad (1)$$

where $C(x,t)$ represents the concentration of chloride ions, as a percentage of weight of concrete at a distance x from the concrete surface, after a time t to be exposed to the source of chloride; and D_c is the diffusion coefficient of chloride.

It is assumed that the corrosion process begins when the chloride concentration at the site where the reinforcement is located, reaches a critical value, C_{cr} ; then, the time in which corrosion starts T_i can be calculated as (Thoft-Christensen, 2001):

$$T_i = \frac{d^2}{4D_c}\left\{erf^{-1}\left(\frac{C_{cr}-C_0}{C_i-C_0}\right)\right\}^{-2} \qquad (2)$$

where C_0 is the balanced concentration of chloride in the concrete surface; C_i is the initial concentration of chloride, both expressed as a percentage of the weight of concrete; d is the coating thickness; and erf is the error function.

2.1.1. Diffusion coefficient

The diffusion coefficient D_c is one of the most important variables in equation 2. In order to make a good approximation of D_c, it is necessary to know the water/cement ratio (w/c) , temperature (Φ) and additives (Jensen et al., 1999). Thoft-Christensen (2001) proposes the following expression for the diffusion coefficient:

$$D_c = 11.146 - 31.025(w/c) - 1.941\Phi + \cdots$$
$$\cdots + 38.212(w/c)^2 + 4.48(w/c)\Phi + .024\Phi^2 \qquad (3)$$

2.2. Start of concrete cracking due to corrosion

The following simple model to determine the reduction of the diameter of the steel reinforcement bars was proposed by Thoft-Christensen (2001):

$$d(t) = d_0 - c_{corr} i_{corr}(t - T_i) \qquad (4)$$

where d_0 is the initial diameter; c_{corr} is the coefficient of corrosion; and i_{corr} is the average annual rate of corrosion.

When the amount of oxide produced is high, it produces pressure on the walls of the concrete around the reinforcing bars. To know the time when the concrete cracking occurs, Thoft-Christensen (2001) proposes the following expression based on studies by Liu and Weyers (1998):

$$\Delta t_{crak} = \frac{W_{crit}^2}{2k_{rust}} \qquad (5)$$

where W_{crit} is the critical mass of rust necessary to produce cracking; k_{rust} is a factor which is proportional to the annual rate of corrosion i_{corr} and the diameter of reinforcement d_0.

$$W_{crit} = \frac{\rho_{steel}}{\rho_{steel} - \alpha\rho_{rust}}\left(W_{porous} + W_{\mathrm{expan}}\right) \qquad (6)$$

$$k_{rust} = 7.039E - 5\left(\frac{1}{\alpha}\right)\pi\ d_0 i_{corr} \qquad (7)$$

$$W_{porous} = \pi\ \rho_{rust} t_{por} d_0 \qquad (8)$$

$$W_{\exp an} = \pi\ \rho_{rust}\left(d_0 - 2t_{por}\right)\ t_{crit} \qquad (9)$$

where ρ_{steel} represents the density of steel; ρ_{rust} is the density of rust; $\alpha = 0.57$ (Liu and Weyers, 1998); W_{porous} is the rust volume required to fill the pore; $W_{\exp an}$ is the volume of corrosion necessary to fill the space due to the concrete expansion; t_{por} is the thickness in the area equivalent to a porosity of one; and t_{crit} is the thickness of the expansion when the fracture starts.

In Eq. 9:

$$t_{crit} = \frac{c\ f_t}{Ec}\left(\frac{a^2 + b^2}{b^2 - a^2} + V_c\right) \qquad (10)$$

$$a = \left(d_0 + 2t_{por}\right)/2 \qquad (11)$$

$$b = c + \left(d_0 + 2t_{por}\right)/2 \qquad (12)$$

where c represents the coating; f_t is the tensile strength of concrete; E_c is the modulus of elasticity; V_c is the Poisson ratio; a is the diameter of the reinforcement; and b is the radial distance from the center of the reinforcement to the coating.

3. STRUCTURAL RELIABILITY ASSESSMENT

In order to evaluate the structural reliability here the concept of average annual rate of structural failure is used, and it is extended to the number of failures in a time interval considering the deterioration (caused by corrosion) of both structural capacity and seismic structural demand for a given intensity.

3.1. Estimating the number of failures in a time interval

The average annual failure rate $\upsilon_F(c)$ represents here the expected number of times per year that the structural capacity (c), associated with a certain limit state, is exceeded due to the effects of the loads corresponding to seismic events of all possible intensities. $\upsilon_F(c)$ can be calculated as follows (Cornell, 1968; Esteva, 1968):

$$\upsilon_F(c) = \int_0^\infty \left|\frac{d\upsilon(y)}{dy}\right| P\left[c < S\,\middle|\,y\right] dy \qquad (13)$$

where $\left|\frac{d\upsilon(y)}{dy}\right|$ is the absolute value of the derivative of the seismic hazard curve; and $P\left[c < S|y\right]$ is the probability that the structural capacity (corresponding to a limit state) c, is smaller than the structural demand, S, given an intensity, y.

The expected number of failures during a time interval is equal to the annual failure rate integrated over a time interval. If the variation of the structural capacity and the seismic demand for a given intensity is considered, then, the expected number of failures evaluated over the time interval $[t, t+\Delta t)$ is expressed as (Tolentino et al., 2012):

$$E\left[\eta_F(t,\Delta t)\right] = \int_t^{t+\Delta t}\int_0^\infty\int_0^\infty\int_0^\infty \left|\frac{d\upsilon(y)}{dy}\right| P\left[C(\tau) < S(\tau)\,\middle|\,y,\tau\right]\cdots$$

$$\cdots f_C(c|\tau)\, f_S(s|y,\tau)\, dy\ dc\ ds\ d\tau \qquad (14)$$

where $P\left[C(\tau) < S(\tau)|y,\tau\right]$ is the conditional failure probability when a given intensity y occurs; $f_C(c|\tau)$ represents the conditional density function of the structural capacity at the instant of time τ; and $f_S(s|y,\tau)$ is the conditional density function of structural demand for a given intensity, y, for an instant τ.

Eq. 14 can be solved by means of numerical integration; however, in this study Eqs. 13 and 14 are solved by means of the simplified approach based on the following assumptions (Cornell et al., 2002): *a*) the environmental hazard curve, $\nu(y)$, is represented around the intensity of interest by the function $\nu(y) = ky^{-r}$; where k and r are parameters that fit the seismic hazard curve; *b*) the structural capacity is assumed lognormal with median $\hat{C}$, and standard deviation of the natural logarithm equal to $\sigma_{\ln C}$; *c*) the function $\hat{D} = a \cdot y^{b}$, represents the median value of the structural demand, $\hat{D}$, around the intensity of interest, a and b are parameters that fit the structural demand curve . It is considered that the structural demand presents a lognormal distribution with standard deviation of the natural logarithm for a given intensity *y*, equal to $\sigma_{\ln D|y}$.

Taking into account assumptions *a*, *b* and *c*, and introducing the contribution of epistemic uncertainties, the expected value of the number of failures, $\eta_F(t,\Delta t)$, at the end of a time interval $[t, t+\Delta t)$ is given by (Torres and Ruiz, 2007):

$$\eta_F(t,\Delta t) = \int_{t}^{t+\Delta t} k\left(y_{\hat{C},t}\right)^{-r} \exp\left[\frac{r^2}{2b(\tau)^2}\left(\sigma^2_{\ln D|y_{\hat{C},\tau}} + \cdots \right.\right.$$

$$\left.\left. \cdots + \sigma^2_{\ln C|\tau} + \sigma^2_{UD|\tau} + \sigma^2_{UC|\tau}\right)\right] d\tau \qquad (15)$$

where

$$y_{\hat{C},\hat{D},\tau} = \left[\frac{\hat{C}(t)}{a(t)}\right]^{\frac{1}{b}} \qquad (16)$$

where $y_{\hat{C},\hat{D},\tau}$ is the environmental load intensity associated with the median capacity, $\hat{C}(t)$, at instant τ; $\sigma^2_{\ln D_{y_{\hat{C}|\tau}}}$ and $\sigma^2_{\ln C|\tau}$ are the variances of the natural logarithms of structural demand for a given intensity, D_y, and the structural capacity, $\hat{C}$, to the limit state of interest, respectively; and $\sigma^2_{UD|\tau}$ and $\sigma^2_{UC|\tau}$ are the variances of the epistemic uncertainties associated with structural demand and structural capacity, respectively.

In order to obtain the expected number of failures that takes into account the variation in time of both structural capacity and structural demand for a given intensity due to corrosion, it is necessary to make the following assumptions (plus the assumptions *a*, *b* and *c* mentioned before):

d) The median of the structural capacity given a state of corrosion, $\hat{C}_{corr}$, varies linearly in time *t*:

$$\hat{C}_{corr}(\tau) = \alpha + \beta \cdot t \qquad (17)$$

e) The median of the structural demand, $\hat{D}$, which consider an state of corrosion in the interval $[0, t)$, is given by:

$$\hat{D}_{corr}(\tau) = (e + f \cdot t) \cdot y^{b} \qquad (18)$$

Taking into account assumptions *a, b, c, d* and *e*, and integrating Eq. 15 within the time interval $[t, t+\Delta t)$, then, the expected number of failures considering the variation of the structural capacity and the structural demand for a given intensity is given by:

$$\eta_{F_{corr}}(t,\Delta t) = k \cdot \left[y_{\hat{C}_{corr},\hat{D}_{corr}|\tau}\right]^{-r} \cdot \exp\left[\frac{r^2}{2b^2} \cdot \left(\sigma^2_{\ln D|y_{\hat{C},t}} + \cdots \right.\right.$$

$$\left.\left. \cdots + \sigma^2_{\ln C|t} + \sigma^2_{UD|t} + \sigma^2_{UC|t}\right)\right] \cdot \Omega_{corr}(t,\Delta t) \quad (19)$$

where

$$\Omega_{corr}(t,\Delta t) = \frac{b}{(b-r)\beta} \cdot [\alpha + \beta \cdot t] \cdot \left[\frac{(e+ft)\cdot\beta}{-f\alpha + e\beta}\right]^{-\frac{r}{b}} \cdots$$

$$\cdots \left[-f\left[X;Y;Z;x(t)\right] + f\left[X;Y;Z;x(t+\Delta t)\right]\right] \cdots$$

$$\cdots \left[1 + \frac{\beta \cdot \Delta t}{\alpha + \beta \cdot t}\right]^{1-\frac{r}{b}} \qquad (20)$$

where $\eta_{F_{corr}}(t,\Delta t)$ is the expected value of the number of failures given a state of corrosion at the end of a time interval $[t, t+\Delta t)$; and $\Omega_{corr}(t,\Delta t)$ is the correction factor for the expected number of failures at the end of a time interval (Tolentino et al., 2012; Tolentino and Ruiz, 2015) which considers the variation of the structural capacity and structural demand, for a given state of corrosion. The implicit hypergeometric function $f[X;Y;Z;x(t)]$ in Eq. 20 is solved as follows (Seaborn, 1991):

$$F(A;B;C;z(t)) = 1 + \frac{AB}{C}z + \frac{A(A+1)\cdot B(B+1)}{2!C(C+1)}z^2\cdots$$

$$\cdots+\cdots = \sum_{n=0}^{\infty}\frac{(A)_n(B)_n}{(C)_n}\frac{z^n}{n!} \quad (21)$$

where

$$A = 1-\frac{r}{b};\; B = -\frac{r}{b};\; C = 2-\frac{r}{b} \quad (22, 23 \text{ and } 24)$$

$$z(t) = \frac{f(\alpha+\beta\cdot t)}{f\alpha - e\beta} \quad (25)$$

$$z(t+\Delta t) = \frac{f(\alpha+\beta(t+\Delta t))}{f\alpha - e\beta} \quad (26)$$

where A, C and z can take real values, and B integer values.

3.2. Confidence factor

The Demand and Capacity Factor Design (DCFD) format assumes that υ_F (annual probability of the performance level not being exceeded) is equal to υ_0 (a performance objective). Here it is considered that the expected number of failures at the end of a time interval $[t, t+\Delta t)$ is equal to a prefixed value of υ_0 multiplied by the time interval Δt ; then, the following condition is established:

$$\eta_F(t,\Delta t) = \nu_0 \cdot \Delta t \quad (27)$$

The number of failures corresponding to a confidence level, x, for a certain time interval is given by (Torres and Ruiz, 2007):

$$\eta_F(t,\Delta t) = \hat{\eta}_F(t,\Delta t)\cdot \exp\left(K_{x|t}\cdot\sigma_{\eta L}\right) \quad (28)$$

where

$$\sigma_{\eta U} = \frac{r}{b}\sqrt{\sigma^2_{UD|t}+\sigma^2_{UC|t}} = \frac{r}{b}\cdot\sigma_{UT|t} \quad (29)$$

Substituting Eqs. 19, 28 and 29 into 27:

$$k\left[\frac{\hat{C}_{corr}(t)}{\hat{D}_{corr}(t)}\right]^{-r}\cdot\exp\left[\frac{r^2}{2b^2}\left(\sigma^2_{\ln D|y_{\hat{C},t}}+\sigma^2_{\ln C|t}\right)\right]\cdots$$

$$\cdots\exp\left(K_{x|t}\cdot\frac{r}{b}\cdot\sigma_{UT|t}\right)\cdot\Omega_{corr}(t,\Delta t)\le\nu_0\cdot\Delta t \quad (30)$$

Solving and separating terms, the following is obtained:

$$\lambda_{conf_{corr}}(t,\Delta t)\le\frac{\phi\cdot\hat{C}_{corr}}{\gamma\cdot\hat{D}^{\nu_0}_{corr}}\cdot\left[\frac{\Omega_{corr}(t,\Delta t)}{\Delta t}\right]^{-\frac{b}{r}} \quad (31)$$

where

$$\hat{D}^{\nu_0}_{corr} = [e+f\cdot t]\cdot\left(y_{\nu_0}\right)^{-\frac{b}{r}} \quad (32)$$

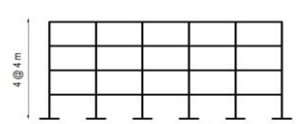
$$\phi = \exp\left[-\frac{r}{2b}\left(\sigma^2_{\ln C,T}+\sigma^2_{CU}\right)\right] \quad (33)$$

$$\gamma = \exp\left[\frac{r}{2b}\left(\sigma^2_{\ln D,T|y}+\sigma^2_{DU}\right)\right] \quad (34)$$

where $\lambda_{conf_{corr}}(t,\Delta t)$ is the confidence factor within the time interval $[t, t+\Delta t)$, which takes into account a state of corrosion.

4. IILUSTRATIVE EXAMPLE

In this section the confidence factor due to corrosion is evaluated for a building located Acapulco, Mexico. It is a 4-story 7-bay reinforced concrete regular structure (see Figure 1). The building was designed in accordance with the Mexico City Building Code (RCDF-2004). A ductility factor Q=3 was used.



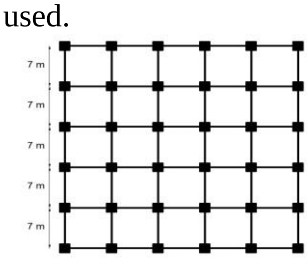


Figure 1: Elevation and plant of the structure analyzed.

4.1. Building Corrosion

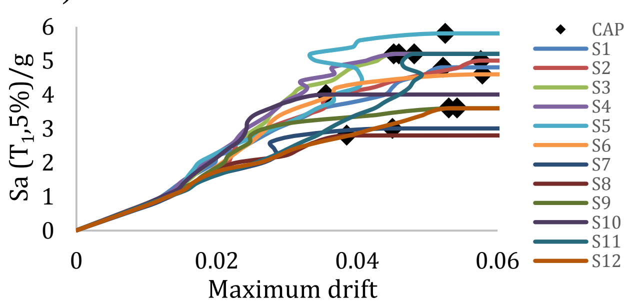
As mentioned in Section 2, there are several intervals of time related to corrosion: the first is the time of corrosion initiation T_i (Eqs. 2 and 3). Assuming the following values $d = 50mm$, $w/c = 0.4$, $\Phi = 27.9°C$, $C_0 = 0.18\%$ (Castañeda *et al.* 1997), $C_{cr} = 0.15\%$ (IMT, No. 182, 2001), and $C_i = 0\%$; then, T_i results equal to 46 years

The second time of interest is when the concrete begins cracking (Δt_{crack}, see Eq. 5). Assuming that: $\rho_{steel} = 3600\,kg/m^3$, $c = 5cm$, $d_0 = 1.9cm$, $\rho_{steel} = 7850\,kg/m^3$, $i_{corr} = 0.9$ (Corrosión Atmosférica, 1999); then, Δt_{crack} results equal to 9 years.

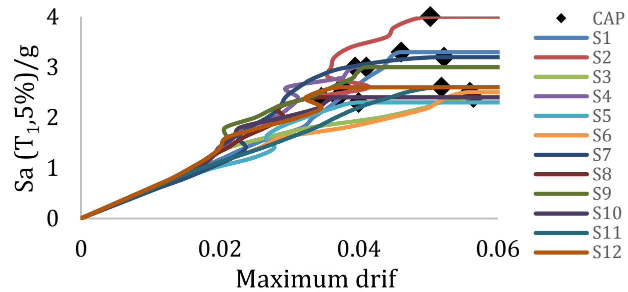
In the present study it was decided to evaluate the reliability of the structure after 46, 55, 75 years, and 100 years of building construction.

4.2. Evaluation of the structural capacity over time

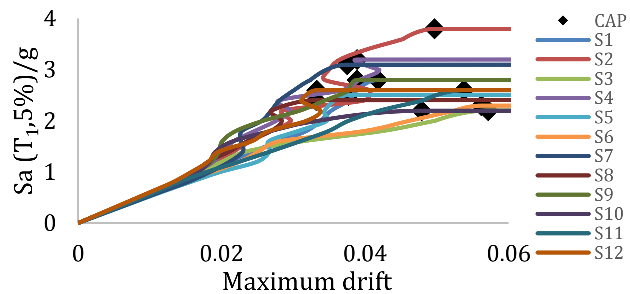
The structural capacity is evaluated here by means of Incremental Dynamic Analysis (IDAs). Figures 2 *a*, *b*, *c* and *d* show the evolution of the structural capacity of the critical story (third story), for each record (S#), corresponding to the following time intervals: 0, 46, 55, 75 and 100 years, respectively. The IDAs were performed using the modified 2D Drain program (Campos and Esteva, 1997).



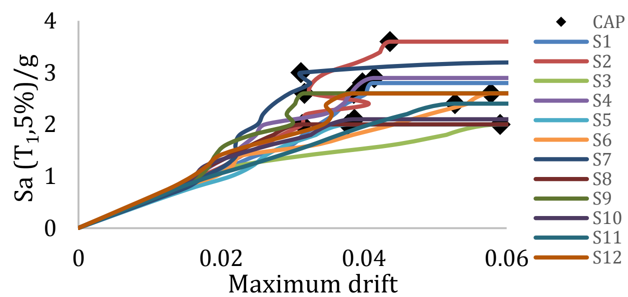
a) 0-46 years



b) 55 years



c) 75 years



d) 100 years

Figure 2: IDAs results for different time intervals.

It is noticed that figures 2 *a*, *b*, *c* and *d* show a reduction in both structural stiffness and structural strength at the end of the time intervals considered, due to the corrosion effect.

The capacity of each structure was obtained considering that the next step in the analysis presents dynamic instability (Vamvatsikos and Cornell, 2002).

The standard deviations of the natural logarithm at the end of the time intervals of interest are equal to 0.15, 0.17, 0.19 and 0.22 for the intervals 0-46, 55, 75 and 100 years, respectively. Figure 3 shows the median value of the structural capacity corresponding to the time intervals of interest.

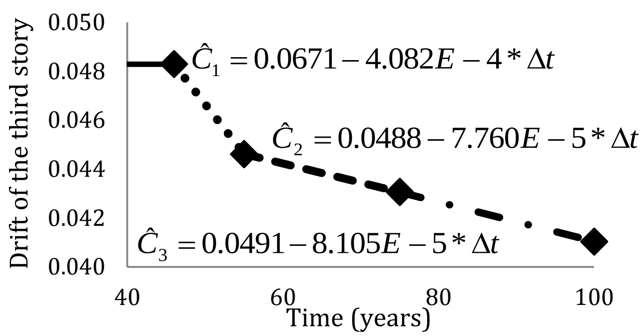
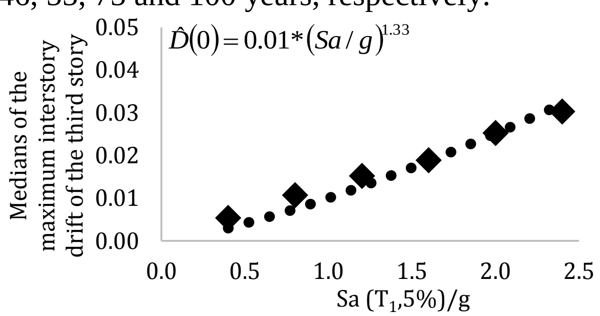


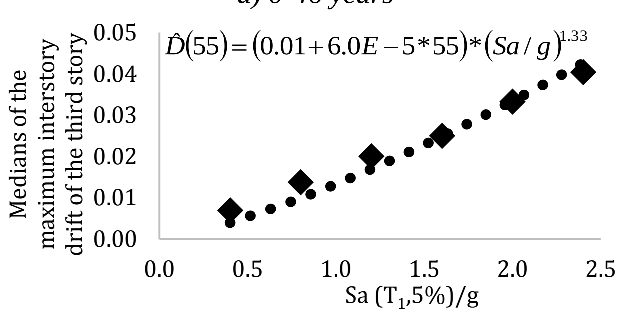
Figure 3: Median of structural capacity, $\hat{C}$, corresponding to different time intervals.

4.3. Evaluation of the structural demand for a given intensity, over time

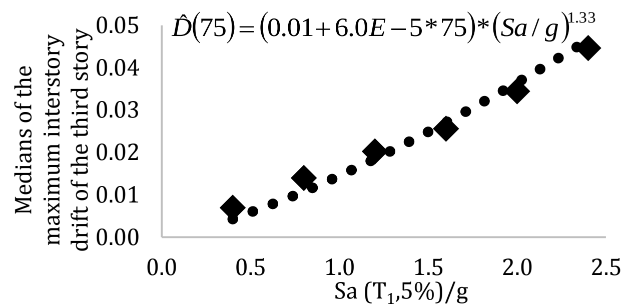
The evaluation of the structural demand was performed by means of a non-linear dynamic analysis "step by step" in time. The structure was subjected to twelve ground motions (S1 to S12). Figures 4 *a, b c* and *d* show the median values of the structural demand corresponding to 0, 55, 75 and 100 years, respectively. The standard deviations of the natural logarithm $\sigma_{\ln D}$ resulted equal to 0.18, 0.21, 0.21 and 0.24 for intervals 0-46, 55, 75 and 100 years, respectively.



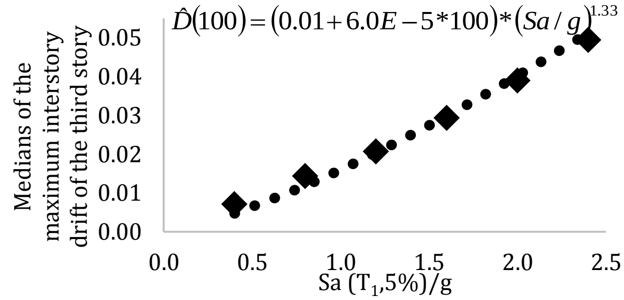
a) 0-46 years



b) 55 years



c) 75 years



d) 100 years

Figure 4: Functions corresponding to the median values of the structural demand.

4.4. Correction factor and expected number of failures over time

The expected number of failures at the end of the time intervals of interest (Eq. 19) is presented in Figure 5. The epistemic uncertainties associated with the structural demand $\sigma_{UD|t}$ and with the structural capacity $\sigma_{UC|t}$ were assumed equal to 0.2. The parameters *k* and *r* were fitted to the seismic hazard curve corresponding to an intensity associated with the near-collapse limit state (story drift limit $\delta = 0.03$).

4.5. Confidence factor over time

Figure 6 shows the confidence factor (λ_{conf}, Eq. 31) normalized with respect to the factor that the structure had when it was constructed $(\lambda_{conf,t=0})$. The figure shows that the behavior of the confidence factor presents the following four major stages:

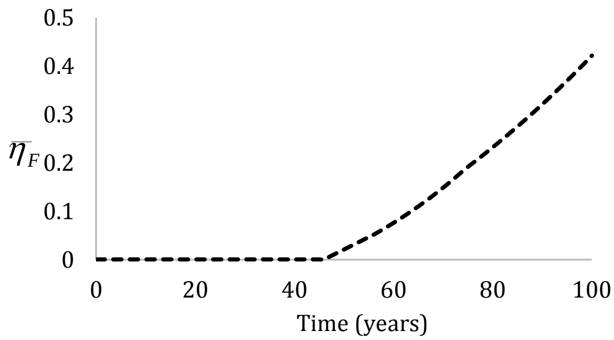


Figure 5: Expected number of failures over time

1) From 0 to 46 years the chloride penetration is presented but it does not compromise the structural reliability; 2) in the second stage, about 46 years after building construction, the steel reinforcement has started corrosion; 3) between 46 and 55 years the degradation of the reinforcing steel and coating cracking in reinforced concrete sections is presented; 4) after 55 years, the section is already cracked and the reinforcement continues corroding.

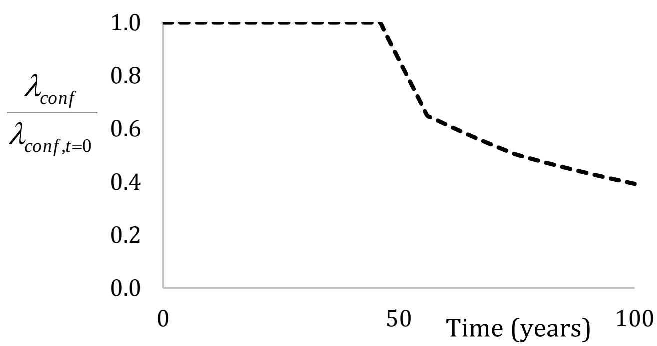


Figure 6: Confidence factor normalized with respect to the initial factor value

5. CONCLUSIONS

It was shown the importance of taking into account the phenomenon of corrosion for reinforced concrete structures that are close to a marine environment, where chloride penetration phenomenon takes place.

For the example shown, the confidence factor of the building was reduced half of its original value after 75 years of building construction.

6. ACKNOWLEDGMENTS

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