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Surface effect on dynamic characteristics of the electrostatically nano-beam actuator☆

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ABSTRACT

A nonlinear pull-in behavior analysis of a cantilever nano-actuator was carried out and an Euler–Bernoulli beam model was used in the examination of the fringing field and the surface and Casimir force effects in this study. In general, the analysis of an electrostatic device is difficult and usually complicated by nonlinear electrostatic forces and the Casimir force at the nanoscale. The nonlinear governing equation of a cantilever nano-beam can be solved using a hybrid computational scheme comprising differential transformation and finite difference to overcome the nonlinear electrostatic coupling phenomenon. The feasibility of the method presented here, as applied to the nonlinear electrostatic behavior of a cantilever nano-actuator, was analyzed. The numerical results for the pull-in voltage were found to be in good agreement with previously published results. The analysis showed that the surface effects had significant influence on the dynamic characteristics of the cantilever nano-actuator.

1. Introduction

In recent years, nano-electromechanical system (NEMS) devices have been widely applied in a diverse range of applications, including nano-switches [1], ultrasensitive sensor [2] and RF communication modules [3] and others, and this certainly makes NEMS research a worthwhile field. When a driving voltage is applied between a moveable and a fixed structure, charges are induced on each which causes an electrostatic force to act on both structures. As the movable structure approaches the fixed structure, an elastic force tends to make it return to its previous undeformed position. At a critical voltage, which is known as the pull-in voltage, instability occurs and the cantilever nano-beam collapses onto the fixed structure. This is of critical importance in the design of NEMS-based devices.

New physics has emerged in the consideration of NEMS. For example, the effect of intermolecular forces, such as the Casimir and van der Waals forces [4], may play an important role at the nano-scale. For separations below 20 nm the force between two surfaces (van der Waals attraction), varies as the inverse cube of the separation. When the separation is greater than 20 nm, the force between two surfaces can be described as the quantum Casimir effect, which is proportional to the inverse fourth power of the separation [5]. Ke et al. [6] calculated the effect of van der Waals force on the pull-in voltage of carbon-nanotubes based NEMS switches. Rotkin [7] considered the effect of the van der Waals force on the pull-in gap and obtained analytical expressions for both the pull-in gap and voltage of a general model. Soroush et al. [4] investigated the effect of dispersion forces on the

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pull-in instability of cantilever nano-actuators using the Adomian decomposition method (ADM). ADM is a practical technique for solving initial value problems (IVP), boundary value problems (BVP), linear, nonlinear and even chaotic systems [8–11].

However, in practice, surface effects such as the residual surface stress and surface elasticity must be taken into account when evaluating the pull-in behavior of NEMs actuators since the large surface area to volume ratio of such structures induces a size-dependent change in many actuator material properties. Ma et al. [12] studied the effects of surface energies and the Casimir force on the instability parameters of cantilever NEMS actuators using the Homotopy perturbation method (HPM). Wang and Feng [13] addressed the effects of both surface elasticity and surface residual stress on the buckling and vibrational behaviur of nanobeams. Fu and Zhang [14] used a modified continuum model to investigate the pull-in behavior of an electrically actuated double-clamped nano-beam incorporating surface effects. In these investigations, the influence of the Casimir attraction as well as fringing field effects was neglected. There is an important pull-in parameter which is the detachment length in the design of NEMS actuators. In the absence of a driving voltage, the Casimir force could overcome elastic restoration and lead to collapse of the cantilever nano-beam. For any fixed initial gap between electrodes, the maximum allowable length of the cantilever nano-beam that will not adhere to the fixed electrode is called the detachment length [12].

Differential transformation theory was first introduced by Pukhov [15,16] to solve linear and nonlinear initial value problems in physical processes. Zhao used the same theory as a means of solving the circuit analysis domain. However, in recent years, the differential transformation method (DTM) has been extended to include a wide range of engineering problems. For example, Chen et al. [17] and Liu et al. [18,19] used DTM to investigate the problem of entropy generation within a mixed convection flow with viscous dissipation effects in a parallel-plate vertical channel. The solution of the fractional telegraph equation has been discussed by Biazar and Eslami [20] who also used DTM. DTM also has been successfully employed to solve many types of linear and nonlinear problems in mathematics [21,22] and engineering, including thermal conduction [23] and optimal control systems [24]. DTM is a powerful semi-analytical tool for general engineering and mechanics problems and yields an analytical solution in the form of a polynomial [22].

In this study, cantilever nano-actuators incorporating two issue effects are investigated in a simulation of dynamic behavior. The first issue considered is that of intermolecular activity such as the Casimir force at the nano-scale. The second is the surface effect. Hence, a governing equation based on the Euler–Bernoulli beam model incorporating nonlinear electrostatic and intermolecular forces has been used in the present study. The rest of this paper has been arranged as follows: Section 2 describes the use of a hybrid computational scheme to complete the nonlinear governing equation of the cantilever nano-beam and specify initial and boundary conditions. Section 3 validates the proposed method by comparison of the numerical results obtained for tip displacement and pull-in voltage of a cantilever nano-beam with the analytical results presented in the literature. Also, the hybrid computational scheme is used to analyze the dynamic response of the cantilever nano-beam as a function of the applied voltage. Finally, Section 4 presents some brief concluding remarks. Compared to existing methods such as the finite difference method, the hybrid numerical scheme has the advantages of an explicit physical meaning and is also simpler and much faster.

2. The cantilever nano-actuator modeling

2.1. Model description

Fig. 1 shows the NEMS cantilever-type actuator considered in the present study. As shown, the cantilever nano-actuator is composed of the cantilever beam of length L with a uniform rectangular cross section of thickness h and width w. The initial gap between the movable beam and the fixed electrodes is denoted by g. In practice, existing NEMS fabrication techniques make separations below 20 nm rather problematical. So, in this study, only the Casimir force is considered. The nonlinear governing equation for the distributed parameter model, based on the Euler–Bernoulli beam assumptions, may be written as [12]

$$(EI)_{eff}\frac{\partial^4 z}{\partial x^4} + \rho A\frac{\partial^2 z}{\partial t^2} = F_{elec} + F_{sur} + F_{cas}, \tag{1}$$

where z represents the deflection of the beam, x denotes the position along the beam axis measured from the clamped end, $(EI)_{eff}$ is the effective bending rigidity of the incorporated surface elasticity effect, ρ is the material density, I in Eq. (1) is the moment of inertia of the cantilever nano-beam and is given by $I = wh^3/12$ and $A = wh$ is the cross-sectional area of the beam. F_{elec} and

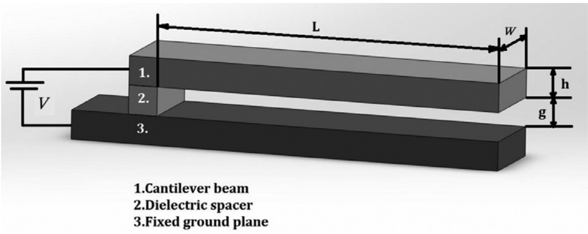

Fig. 1. The cantilever nano-beam system.

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F_{sur} is the electrostatic force, the distributed transverse force resulting from surface effect and F_{cas} is the Casimir force per unit length of the nano-beam.

Considering the first-order fringing field correction of the electrostatic force per unit length of the beam is [25]

$$F_{elec} = \frac{\varepsilon_0 w V^2}{2(g-z)^2}\left(1 + 0.65\frac{g-z}{w}\right), \tag{2}$$

where ε_0 is the permittivity of free space and V is the driving voltage.

Based on the composite beam theory and the assumption that the thickness of the surface layer is much smaller than the beam thickness h, the effective bending rigidity $(EI)_{eff}$ for the nano-beam with rectangular cross section is derived as [12]

$$(EI)_{eff} = EI + \frac{1}{2}E^s wh^2, \tag{3}$$

where E is the corresponding elastic modulus of the material and E^s is the surface modulus of elasticity. The transverse force along the longitudinal direction of the rectangular nano-beam is

$$F_{sur} = 2\tau^0 w\frac{d^2z}{dx^2}, \tag{4}$$

Eq. (4) indicates that residual surface stress comes into effect once the nano-beam is bent with a non-zero curvature. And τ^0 is the residual surface stress in the longitudinal direction. The Casimir force per unit length of the nano-beam is

$$F_{cas} = \frac{\pi^2\hbar c w}{240(g-z)^4}, \tag{5}$$

where $\hbar$ is Planck's constant (1.055×10^{-34} J) divided by 2π and c is the speed of light (2.998×10^8 m/s^2). Taking into account the fringing field effect, the Casimir force effect, and the residual surface stress and surface elasticity, the nonlinear governing equation for the cantilever nano-beam shown in Fig. 1 is given as

$$(EI)_{eff}\frac{\partial^4 z}{\partial x^4} + \rho A\frac{\partial^2 z}{\partial t^2} = \frac{\varepsilon_0 w V^2}{2(g-z)^2}\left(1 + 0.65\frac{g-z}{w}\right) + 2\tau^0 w\frac{d^2z}{dx^2} + \frac{\pi^2\hbar c w}{240(g-z)^4}. \tag{6}$$

The boundary conditions of the cantilever nano-beam are given by

$$\begin{aligned} z(x,t) &= \frac{\partial z(x,t)}{\partial x} = 0 \qquad \text{at } x = 0,\\ \frac{\partial^2 z(x,t)}{\partial x^2} &= \frac{\partial^3 z(x,t)}{\partial x^3} = 0 \qquad \text{at } x = L. \end{aligned} \tag{7}$$

Finally, the initial condition of the cantilever nano-beam is defined as

$$z(x,0) = \frac{\partial z(x,0)}{\partial t} = 0. \tag{8}$$

2.2. Dimensionless nonlinear governing equation

For analytical convenience, the transverse displacement of the nano-beam z is normalized with respect to the initial gap between the electrodes, the axial length position x is normalized with respect to the nano-beam length, and the time t is normalized with respect to the time constant $\bar{T}$, where $\bar{T}$ is defined as $\bar{T} = \sqrt{\rho A L^4/(EI)_{eff}}$. In addition, the width of the nano-beam w is normalized with respect to the initial gap between the electrodes, i.e.

$$\bar{z} = \frac{z}{g}, \quad \bar{x} = \frac{x}{L}, \quad \bar{t} = \frac{t}{\bar{T}}, \quad \bar{w} = \frac{w}{g}. \tag{9}$$

Let the following parameters also be defined:

$$\begin{aligned} \bar{\tau} &= \frac{2\tau^0 w L^2}{(EI)_{eff}}, \quad \bar{F}_1 = \frac{w\varepsilon_0 L^4}{2g^3(EI)_{eff}},\\ \bar{F}_2 &= \bar{F}_1 \times \left(\frac{0.65 \times g}{\bar{w}}\right), \quad \bar{F}_3 = \frac{\pi^2\hbar c w L^4}{240(EI)_{eff} g^5}. \end{aligned} \tag{10}$$

Substituting Eq. (9) into Eqs. (6), (7) and (8). The dimensionless nonlinear governing equation of the cantilever nano-beam is obtained as

$$\frac{\partial^4 \bar{z}}{\partial \bar{x}^4} + \frac{\partial^2 \bar{z}}{\partial \bar{t}^2} - \bar{\tau}\frac{\partial^2 \bar{z}}{\partial \bar{x}^2} = \frac{\bar{F}_1 V^2}{(1-\bar{z})^2} + \frac{\bar{F}_2 V^2}{1-\bar{z}} + \frac{\bar{F}_3}{(1-\bar{z})^4}. \tag{11}$$

where the dimensionless parameters $\bar{F}_1$, $\bar{F}_2$ and $\bar{F}_3$, account for the electrostatic force, fringing effect and the Casimir force. The first, second and third terms on the right-hand side of Eq. (11) can be approximated by a Taylor series expansion. In other

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Table 1
Comparison of present analytical results and literature results via applied voltage 1 V.

Thickness (nm)	Non-dimensional end-gap deflection		Error (%)
	Hybrid computational scheme (A)	Finite difference method (FDM) [12] (B)	Δe_1
50	0.008572	0.007986	7.34
60	0.004557	0.004366	4.37
70	0.002732	0.002671	2.28
80	0.001773	0.001761	0.68
90	0.001218	0.001225	0.57
100	0.000874	0.000887	1.47

$^{*}\Delta e_1 = \frac{|(A)-(B)|}{(B)} \times 100\%$

words, neglecting terms higher than the fourth order, the nonlinear governing equation of the cantilever nano-beam can be reformulated as

$$\frac{\partial^4 \bar{z}}{\partial \bar{x}^4} + \frac{\partial^2 \bar{z}}{\partial \bar{t}^2} - \bar{\tau}\frac{\partial^2 \bar{z}}{\partial \bar{x}^2} = V^2\left[\bar{F}_1\left(1+2\bar{u}+3\bar{u}^2+4\bar{u}^3\right) + \bar{F}_2\left(1+\bar{u}+\bar{u}^2+\bar{u}^3\right)\right] + \bar{F}_3\left(1+4\bar{u}+10\bar{u}^2+20\bar{u}^3\right). \tag{12}$$

The corresponding boundary conditions are given by

$$\bar{z}(\bar{x},\bar{t}) = \frac{\partial \bar{z}(\bar{x},\bar{t})}{\partial \bar{x}} = 0 \quad \text{at } \bar{x}=0,$$
$$\frac{\partial^2 \bar{z}(\bar{x},\bar{t})}{\partial \bar{x}^2} = \frac{\partial^3 \bar{z}(\bar{x},\bar{t})}{\partial \bar{x}^3} = 0 \quad \text{at } \bar{x}=1. \tag{13}$$

Finally, the initial condition is defined as

$$\bar{z}(\bar{x},0) = \frac{\partial \bar{z}(\bar{x},0)}{\partial \bar{t}} = 0. \tag{14}$$

The dimensionless parameters are $\bar{F}_1, \bar{F}_2, \bar{F}_3$. Having transformed the governing equation and its associated boundary conditions and initial condition in the time domain, the finite difference approximation method is applied with respect to $\bar{x}$, i.e. the normalized axial position along the length of the nano-beam [17–19].

3. Numerical results and discussion

To start with the validity of the hybrid computational scheme derived in the previous section was verified by analyzing the cantilever nano-beam involved in the nonlinear governing equation. The computations were performed using MATLAB and a [001] silver cantilever nano-beam of length 1 μm, width $w = 5 \times h$, initial gap $g = 50\,nm$, and a Young's modulus $E = 76\,GPa$ with different values of beam thickness h, ranging from 50 to 100 nm. The residual surface stress τ^0 and surface modulus of elasticity E^s are 0.89 μN/μm and 1.22 μN/μm, respectively. The end deflections of the cantilever nano-beam obtained using the two approaches are listed in Table 1 for comparison, with an applied voltage of 1 V. It can be seen that the hybrid computational scheme prediction lies close to that of the Finite Difference Method (FDM), and the maximum difference between the two predictions is 7.38% when $t = 50\,nm$. Table 2 compares the results obtained by the hybrid computational scheme for the pull-in voltage under a driving voltage obtained by the HPM presented in [12]. As shown, in the absence of surface effect, the hybrid computational scheme predicts a pull-in voltage of 5.91 V. By contrast, the HPM predicts a pull-in voltage of 5.588 V. In other words, the pull-in voltage computed using the hybrid computational scheme deviates by no more than 5.76% from the value presented in the literature.

Fig. 2 shows the variation of the non-dimensional tip deflection with the residual stress (τ^0) as a function of the applied voltage. Note that the actuating conditions are as follows: different values of the applied voltage range from 3 to 5 V, $E^s = 1.22\,\mu$N/μm. It can be seen that for a constant applied voltage, the residual stress increases with increasing tip deflection. In addition, it is seen that for a constant residual stress, the tip deflection increases with increased voltage due to the resulting rise in electrostatic force.

Table 2
Comparison of results obtained for pull-in voltage without surface effect.

Numerical method	Pull-in voltage (V)
Homotopy perturbation method (HPM) [12] (A)	5.588
Hybrid computational scheme (B)	5.91
Error (%)	$\Delta e_2 = 5.76$

$^{*}\Delta e_2 = \frac{|(A)-(B)|}{(A)} \times 100\%$

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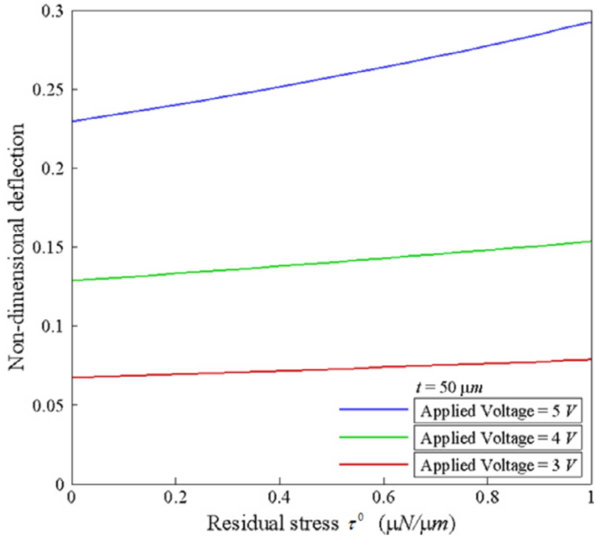

Fig. 2. Variation of the non-dimensional tip deflection with residual stress (τ^0) as a function of the applied voltage (thickness = 50 nm).

Fig. 3 illustrates the deflection of the cantilever nano-beam at various residual stress (τ^0). The modeling parameters are $L = 1\,\mu$m, width $w = 5 \times h$, initial gap $g = 50\,nm$, Young's modulus $E = 76\,GPa$, the beam thickness $h = 30\,nm$, and the surface elastic modulus $E^s = 0\,\mu$N/μm with different values of residual surface stress τ^0, ranging from −0.5 to 0.5 μN/μm. Note that in every case, the nano-beam is actuated by an applied voltage of 1 V. The results indicate that the non-dimensional tip deflection increases as the residual surface stress is increased from a negative to a positive value.

Fig. 4 shows the influence of surface effects on variation of the detachment length with beam thickness. The initial gap g for the nano-beam is taken as 30 nm and the beam width is $w = 5 \times h$. The residual surface stress τ^0 and surface modulus of elasticity E^s are 0.89 μN/μm and 1.22 μN/μm, respectively. The results indicate that for a constant detachment length, the beam thickness and surface effect are linked and it can be seen that the influence of surface effects decreases as the beam thickness goes up.

Fig. 5 shows the transverse beam deflection along the axial direction given actuation voltages ranging from 2 to 2.2 V. For a case where the thickness $h = 30\,nm$, the initial gap g for the nano-beam is taken as 50 nm while the beam width $w = 5 \times h$ and length is $L = 1\,\mu$m. As expected, the beam deflection increases with an increasing distance from the clamped end for all voltages considered. The nano-beam remains stable for all actuation voltages less than 2.1 V, but pull-in occurs at an actuation voltage of 2.2 V.

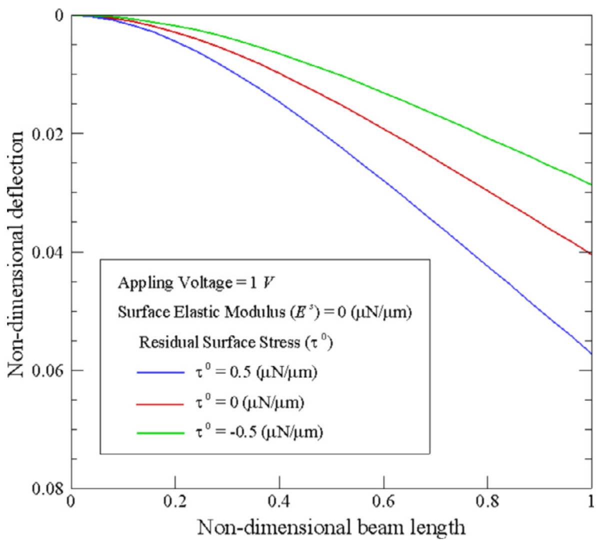

Fig. 3. Variation of non-dimensional tip deflection along non-dimensional beam length as a function of residual stress (τ^0) (thickness = 30 nm and the surface elastic modulus $E^s = 0\,\mu$N/μm).

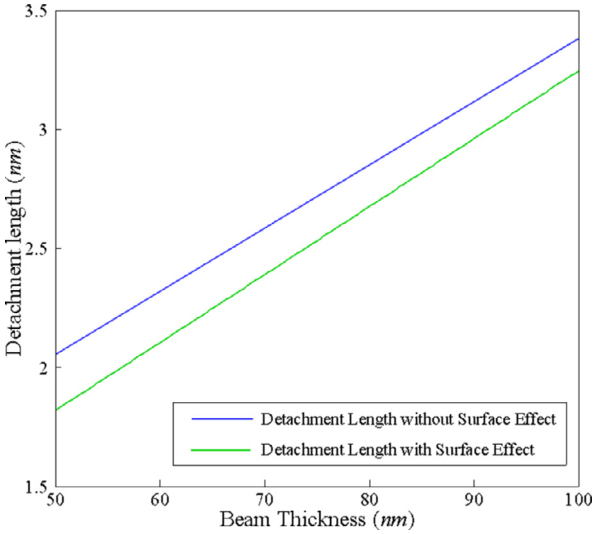

Fig. 4. Influence of surface effects on variation of the detachment length with beam thickness (thickness = 50 nm).

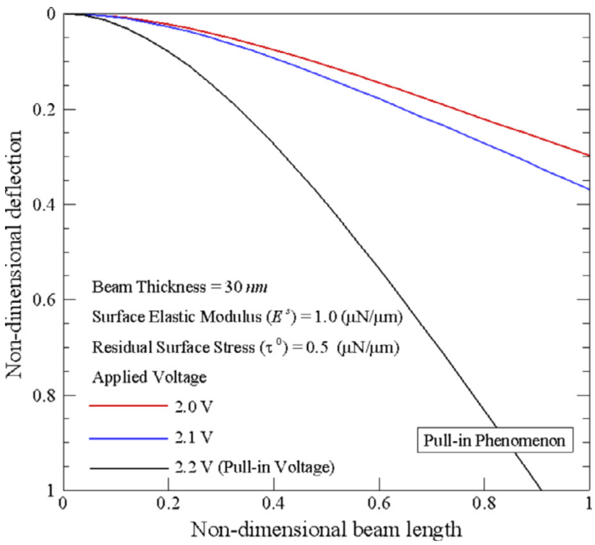

Fig. 5. Variation of non-dimensional transverse deflection along non-dimensional beam length as a function of applied voltage.

4. Conclusions

A hybrid computational method employing differential transformation and finite difference approximation was used in this study to analyze the nonlinear behavior of an electrostatically actuated cantilever nano-beam. In the computational method, each term in the non-dimensional governing equation of the cantilever nano-beam is processed using the differential transformation method with respect to the time domain t. The transformed equation is then processed using finite difference approximation with respect to $\bar{x}$ (i.e. the axial direction of the cantilever nano-beam). The validity of the hybrid computational scheme has been confirmed by comparing the numerical results for pull-in voltage of the cantilever nano-beam with the results obtained using the HPM. The analysis shows that surface effects significantly influence the dynamic characteristics of the cantilever nano-actuator. The proposed method has been used to examine the surface effects of the nano-beam and beam thickness on the detachment length without voltage actuation. In addition, it has been shown that the dynamic response of the nano-beam can be confined to a stable operating region given an appropriate setting of the actuation voltage. Overall, the results presented in this study confirm the usefulness of the proposed hybrid computational scheme to model complex interactions between electrostatic coupling, the fringing field, the Casimir force and the surface effects. As a result, this is presented as a suitable technique for analyzing not only the nonlinear response of the cantilever nano-beam considered in this study, but also that of other nano-structural devices.

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