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RESEARCH ARTICLE - SYSTEMS ENGINEERING

A Generalized Economic Order Quantity Inventory Model with Shortage: Case Study of a Poultry Farmer

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Abstract

We consider an EOQ inventory model for growing items, wherein the value and size of items increase during time, some instances of these items are livestock, fish, and poultry. The main difference between this inventory system and older ones is weight increment of products during stocking without buying more. This paper studies an inventory system of poultries that new-born items are fed to reach the ideal weight for consumers. In this study, based on the consumers' preference of fresh foods over frozen items, we assume that shortage is permitted and consumers wait for fresh items when company pays some additional penalties, i.e., the shortage is fully backordered. On the other hand, for each cycle, the producer must prepare the place in terms of hygiene conditions; thus, a setup time per cycle is considered. The aim of this study is to obtain optimum system solution, such that total costs, including setup, purchasing, holding, feeding, and shortage, are minimized. To do so, we employ mathematical measures to approximate growing rates and model the system as a non-linear programming. To solve the obtained optimization model, we employ hessian matrix to obtain optimal solution for this inventory system. The proposed EOQ inventory model helps poultry industries in Iran to optimize their system considering costs and permissible shortage, and it can be employed in other countries. Finally, we provide a numerical example and its sensitivity analysis, plus some potential future directions.

Keywords Inventory management · Economic order quantity · Growing items · Non-linear programming · Permissible shortage · Poultry

1 Introduction

Trying to optimize organization costs by managing inventories goes back to more than a century ago when the first economic order quantity (EOQ) inventory model was proposed by Harris [1]. Harris's inventory model minimizes total costs, including holding and ordering costs, such that inventory system faces no shortages. An important modification of EOQ inventory model is the economic production quantity (EPQ) model proposed by Taft [2], where instead of receiving products in orders at once; they are produced at a known rate.

Recently, Rezaei [3] investigated an EOQ inventory model for products that are growing during storage, for industries such as livestock, fish farming, and poultry. In these inventory systems, the weight of products increases during the period of stocking without ordering additional items. The aforementioned study is the first systematic research that considers this class of inventory. To do so, it develops a general inventory model then extends that inventory model for poultry. After this study, Zhang et al. [4] developed the aforementioned inventory model for cases that legislator imposes a constraint carbon emission or put penalty for it.

Before these studies, the EOQ inventory model has been modified by several researchers and academicians to relax some particular boundaries of products specifications. One

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of the first attempts to model inventory systems for unconventional items dates back to Whitin [5], who addressed goods that became old-fashioned after a specified period. Afterwards, Ghare and Schrader [6] studied an inventory system, wherein items decayed exponentially. Then, Covert and Philip [7] extended Ghare and Schrader [6] study for cases that deterioration rate could vary during time. Other studies that dealt with relaxing infinite life cycle and modifying the model for perishable products such as vegetable, dairies, batteries, and drugs are: Muriana [8]; Dobson et al. [9]; Yan and Wang [10]; and Boxma et al. [11]. For comprehensive reviews about inventory models of perishable items, see Goyal and Giri [12] and Bakker et al. [13].

On the other hand, some other studies extended the EOQ/EPQ inventory models for imperfect products, for example, Rosenblatt and Lee [14] considered a production system that after some time the manufacturing system alters to out of control and starts to produce imperfect items. Afterwards, Salameh and Jaber [15] studied an EOQ/EPQ inventory model with imperfect quality items. These items were either suitable for other processes or could be sold in a batch after inspection process. Hayek and Salameh [16] investigated the rework of produced imperfect quality items in an imperfect production system for an EPQ inventory model. Moreover, some research addressed EOQ/EPQ model when the received batch should be examined to identify defective items. Examples are Manna et al. [17], Mukhopadhyay and Goswami [18], Nobil et al. [19] and Pasandideh et al. [20].

Furthermore, some researchers extended classical EOQ inventory model considering trade credits policies to overcome capital boundaries of system. First kind of these studies is Goyal [21], wherein purchasing costs could be paid back with a permissible delay. In this model, the supplier permits the vendor to pay part of the costs during the period. Then, Rajan and Uthayakumar [22] developed an EOQ inventory model with permissible delays in payment for cases that demand and holding costs change as a function of time. In another study, Pasandideh et al. [23] studied trade credits for an EOQ inventory model considering several items, permissible shortage, and warehouse constraint using genetic algorithm.

Another way to overcome system costs in inventory models is by decreasing inventory level considering permissible shortage. Using this, system managers avoid investing too much on building warehouses, especially for items such as food that requires temperature regulation, hygiene, and other special conditions. One of the first kinds of these studies was performed by Hadley and Whitin [24]. They revised Haris [1] model for cases that system faces some shortages. San-José et al. [25] developed an EOQ inventory model considering partial backordering, i.e., some of the customers do not wait for new items, considering non-linear holding costs. Pervin et al. [26] addressed an inventory system with permissible shortage, random deterioration, and time-dependent demand and holding cost to obtain optimum replenishment policy.

This paper considers an inventory system for a single product. By suitable nutrition, new-born items grow and reach the ideal weight for satisfying customers demand. It is assumed that shortage is permitted. On the other hand, a setup time is considered to prepare nurturing environment before a new cycle. The aim of this study is to determine the optimum values of shortage and cycle-length subject to minimizing inventory system total costs including setup, purchasing, holding, feeding, and shortage. Moreover, the mathematical model of this inventory system has a non-linear programming (NLP) form. In "Appendix A", it is proven that this NLP is a convex problem. Furthermore, an exact solution algorithm for this problem is proposed by employing convexity property of the model. The proposed EOQ inventory model with growing items is applicable in instances that the growing function estimation with a linear function does not introduce unacceptable errors to the system. The proposed EOQ inventory model helps livestock, poultry, and aquaculture industries to optimize their inventory system considering feeding costs, growing rate, and improvement of revenue management. Using these analytical approaches, industry managers can make the optimal decisions about ordering time and purchasing new-born items, growing and shortage period length. This model developed based on historical data so is more beneficial than decision making by a conjecture or trial and error.

Considering permissible shortage in the proposed model makes it closer to real-world scenarios and helps managers to calculate optimum shortage period when face flexible customers, i.e. their customers do not supply their demand from other suppliers in case of shortage. Moreover, in most poultry and aquaculture cases, a setup time is required for system inspection and preparation for another run. Finally, proposing a straightforward solution procedure and a linear estimation of growth function helps managers to obtain a near optimum solution. A comparison between this work and former studies is proposed in Table 1.

The rest of this paper is organized as follows. Section 3 provides the notation, assumptions, and problem definition. Section 4 presents an exact algorithm for solving the EOQ inventory model with growing items. Section 5 solves a numerical example with proposed algorithm and also presents a sensitivity analysis of the inventory model. Finally, Sect. 6 presents some conclusions and potential future research directions.

Table 1 Comparison among popular EOQ inventory models, models considering growing items, and proposed inventory model

Research	Product				Shortage	System			Trade credit	Constraints	Solution method	
	Fix	Perishable	Deterioration	Growing		Ordering	Producing	Growing			Closed-Form	Heuristic
Harris [1]	*					*					*	
Taft [2]	*						*			*	*	
Hadley and Whitin [24]	*				*	*					*	
Goyal [21]	*					*			*		*	
Ghare and Schrader [6]		*				*						*
Salameh and Jaber [15]			*			*					*	
Hayek and Salameh [16]			*		*		*				*	
Rezaei [3]				*				*				*
Zhang [4]				*				*		*		*
This paper				*	*			*		*	*	

2 Contributions and Assumptions

In this section, we discuss the contributions of this paper. Former studies [3,4] in growing items employed Richards' growth function; this approach limits the optimization procedure by increasing the complexity of modelling and calculations. Moreover, those studies assume that consumer would not wait for items arrival in case of unavailability, i.e. shortage was not permitted. This study relaxes those assumptions and makes studying of growing items much easier. To do so, first, we assume that growing rate of items can be approximate by linear functions. Then, this assumption is assessed using a case study of an Iranian poultry producer. Moreover, we considered consumer preference by addressing (a) the preferred weight of chicken by consumers and (b) the tolerance of delay in delivery of fresh chicken if company pays some penalty fee. The first assumption is crucial in practice, since optimizing the system without consumer consideration may encourage the managers to produce overweight or underweight items, especially in a market like Iran that consumers prefer to buy uncut chickens, and this cause losing customers. Moreover, several studies indicate that the consumer prefers chilled or fresh products over frozen ones. For instance, Vukasovič [27] studied three European countries preferences, namely, Slovenia, Bosnia and Herzegovina, and Serbia; the findings suggest that main question of consumers about poultry was "is fresh". This attitude towards fresh poultry paves the way for managers to decrease system costs that have a direct effect on product price. Moreover, to maintain satisfaction of the consumers who waited for a period of time before delivery of fresh items, a latency penalty is considered. Finally, the procedure of this research is proposed by a graphical diagram. Figure 1 represents a schematic overview of the system and the solution procedure.

3 Problem Definition

The inventory problem considers breeding new-born animals for food by optimizing the amount of initial purchase and slaughtering time with respect to market demand. In this inventory model, product shortage is permitted and fully backordered. Additional assumptions of the proposed EOQ inventory model with growing items are as follows:

- There is an additional cost for feeding and raising items.
- Items growth can be approximated by a linear function.
- Feeding cost is directly related to gained weight.
- The holding cost is calculated at the moment items gained their final weight and are ready to slaughter.

The following notation is employed for mathematical formulation:

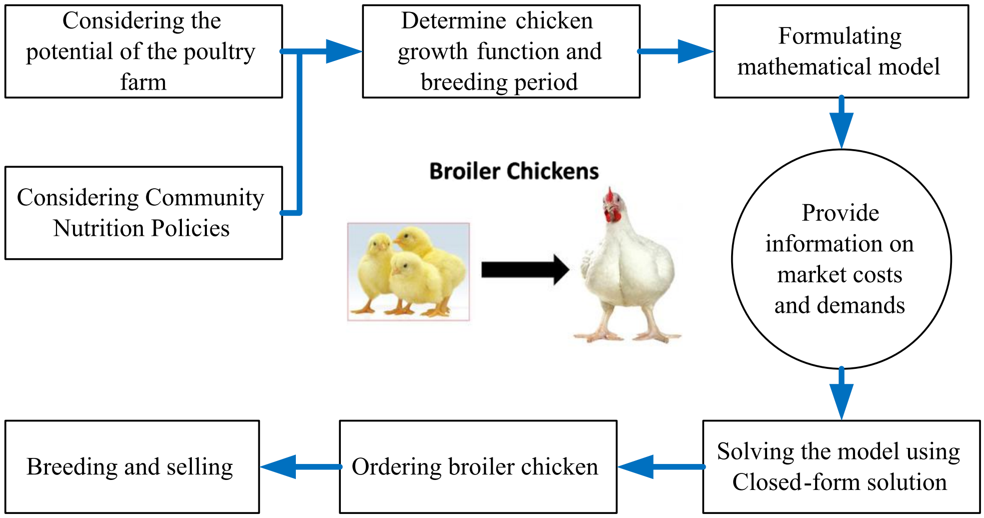

Fig. 1 Schematic overview of system study and solution procedure

D : Demand rate per time unit $\left(\frac{\text{gr}}{\text{year}}\right)$.
k : Growing rate per chick per time unit $\left(\frac{\text{gr}}{\text{chick}\times\text{year}}\right)$.
w_0 : Approximated weight of new-born items (gr).
w_1 : Approximated weight at the moment of slaughtering (gr).
Q_t : Total weight of inventory at t.
t_1 : Growing period.
t_2 : Consumption period.
t_3 : Shortage period.
T : Length of each period (decision variable).
y : Number of ordered items per period.
S : Shortage quantity per period (gr).
c : Purchasing cost per weight unit $\left(\frac{\$}{\text{gr}}\right)$.
r : Feeding cost per weight unit per time unit $\left(\frac{\$}{\text{gr}\times\text{year}}\right)$.
h : Holding cost per weight unit per time unit $\left(\frac{\$}{\text{gr}\times\text{year}}\right)$.
f : Shortage cost per weight unit per time unit $\left(\frac{\$}{\text{gr}\times\text{year}}\right)$.
A : Setup cost (ordering cost), cost of preparing nurturing environment per period $\left(\frac{\$}{\text{setup}}\right)$.
ts : Setup time (unit time), time of preparing nurturing environment per period $\left(\frac{\text{year}}{\text{setup}}\right)$.

diagram of inventory on hand for un-growing items and growing items are proposed in Figs. 2 and 3, respectively.

Figure 2 proposes an inventory system for conventional items like books, frozen food, and other items, where the supplier buys them and stock them for an amount of time, t_1; then, items are sold at a rate, i.e., demand rate D, until the inventory level reaches zero. Finally, during t_3, the inventory system faces shortage until the time that the amount of shortage reaches S.

In Fig. 3, y is the quantity of purchased items for a period from outside suppliers and w_0 and w_1 are approximated weights at the beginning of the period (new-born chicks) and end of the period (ready to sell product), respectively. Consequently, total initial and final weights are $Q_0 = yw_0$ and $Q_1 = yw_1$, respectively. Knowing that, t_1 is the period that the animals are raised, and afterwards, they are sold during t_2. Moreover, t_3 is a period of time that inventory system faces S units of shortage. In this inventory problem, at the beginning of consumption period S, immediate demands must be satisfied, so inventory level diminishes as $Q_2 = yw_1 - S$. Based on these, the following equations are established:

$$t_1 = \frac{Q_1 - Q_0}{yk} = \frac{w_1 - w_0}{k} \tag{1}$$

$$t_2 = \frac{Q_2}{D} = \frac{yw_1 - S}{D} \tag{2}$$

$$t_3 = \frac{S}{D} = \frac{S}{D}. \tag{3}$$

Therefore, cycle length is calculated as

$$T = t_2 + t_3 = \frac{yw_1 - S}{D} + \frac{S}{D} = \frac{yw_1}{D}. \tag{4}$$

Hence

$$y = \frac{DT}{w_1}. \tag{5}$$

The objective of this inventory model is to optimize total cost per period (TCU) including purchasing cost (BC), holding cost (HC), operational costs (OC), shortage cost (SC), and food procurement cost (PC). In other words

$$\text{TCU} = \text{BC} + \text{HC} + \text{OC} + \text{SC} + \text{PC}. \tag{6}$$

The following subsection represents a detailed discussion of calculation of aforementioned costs.

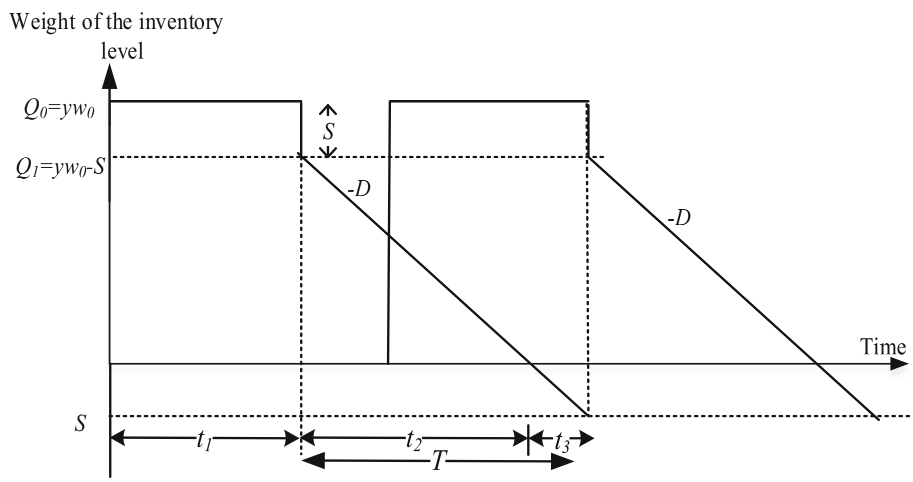

Fig. 2 Diagram of the weight of the inventory on hand for conventional items with shortage

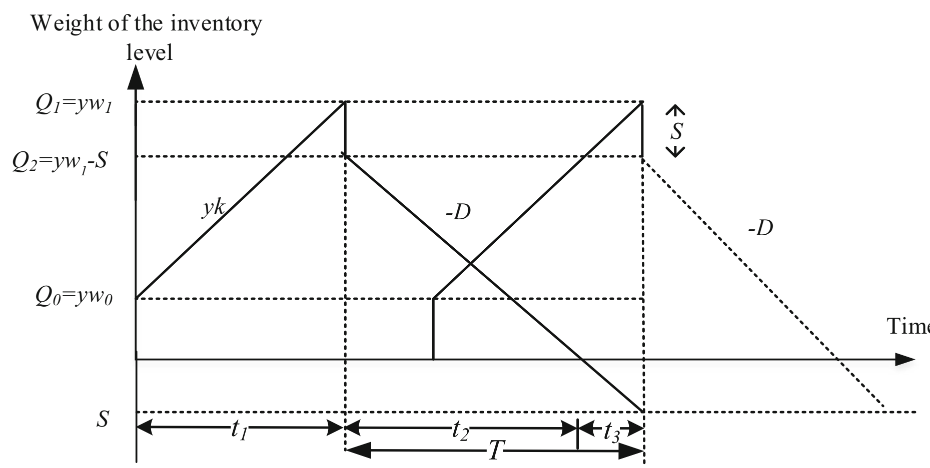

Fig. 3 Diagram of the weight of the inventory on hand for growing items with shortage

3.1 Purchasing Cost Per Period

The price per gram is c, so the total purchasing cost per period obtained by Eq. (7):

$$\mathrm{BC} = cyw_0. \quad (7)$$

By substituting y from Eq. (5), we have

$$\mathrm{BC} = cw_0\left(\frac{DT}{w_1}\right) = \frac{Dcw_0T}{w_1}. \quad (8)$$

3.2 Holding Cost Per Period

Holding cost per gram per time unit is h. Moreover, holding cost is calculated at the end of feeding period (t_2). Based on Fig. 3, holding cost per period is expressed as

$$\mathrm{HC} = h\left(\frac{t_2Q_2}{2}\right) = h\left(\frac{(yw_1 - S)^2}{2D}\right) = \frac{h}{2D}\left[(yw_1)^2 + (S)^2 - 2yw_1S\right]. \quad (9)$$

By substituting y from Eq. (5), we have

$$\mathrm{HC} = \frac{hS^2}{2D} - hTS + \frac{hDT^2}{2}. \quad (10)$$

3.3 Operational Costs Per Period

Operational cost per growing period is A, so we have

$$\mathrm{OC} = A. \quad (11)$$

3.4 Shortage Cost

Shortage cost per gram per time unit is f, so based on Fig. 3, total shortage costs can be calculated as

$$\mathrm{SC} = f\left(\frac{t_3S}{2}\right) = f\left(\frac{(S)^2}{2D}\right). \quad (12)$$

3.5 Food Procurement Per Period

Food procurement cost per gram per time unit is r, with respect to Fig. 3 food procurement costs per period, is calculated as

$$\mathrm{PC} = r\frac{t_1\left(Q_1 - Q_0\right)}{2} = r\left(\frac{y\left(w_1 - w_0\right)^2}{2k}\right). \tag{13}$$

By substituting y from Eq. (5), we have

$$\mathrm{PC} = \frac{Dr\left(w_1 - w_0\right)^2 T}{2kw_1}. \tag{14}$$

With regard to Eqs. (8), (10–12), and (14), the inventory total costs per period are formulated as follows:

$$\begin{aligned}\mathrm{TCU} &= \mathrm{BC} + \mathrm{HC} + \mathrm{OC} + \mathrm{SC} + \mathrm{PC}\\ &= \frac{Dcw_0T}{w_1} + \frac{hS^2}{2D} - hTS + \frac{hDT^2}{2}\\ &\quad + A + f\left(\frac{(S)^2}{2D}\right) + \frac{Dr\left(w_1 - w_0\right)^2 T}{2kw_1}.\end{aligned} \tag{15}$$

Then, inventory total cost per time unit is computed as follows:

$$\begin{aligned}\mathrm{TC} = \frac{\mathrm{TCU}}{T} &= \frac{Dcw_0}{w_1} + \frac{Dr\left(w_1 - w_0\right)^2}{2kw_1}\\ &\quad + A\left(\frac{1}{T}\right) + \left(\frac{h+f}{2D}\right)\left(\frac{(S)^2}{T}\right) + \frac{hD}{2}(T) - h(S).\end{aligned} \tag{16}$$

3.6 Constraint

In the proposed inventory system to guarantee that items are ready to use on time, the total setup and growth time should be less than or equal to consumption and shortage period. Therefore, the following constraint must be satisfied:

$$t_1 + ts \leq T. \tag{17}$$

Substituting, t_1 from Eq. (1), obtains the following constraint:

$$\frac{w_1 - w_0}{k} + ts \leq T. \tag{18}$$

By substituting y from Eq. (5) transforms, the above-mentioned constraint into the following for production period:

$$T \geq \left\{\frac{w_1 - w_0}{k} + ts = T_{\min}\right\}. \tag{19}$$

3.7 Mathematical Formulation of the EOQ Inventory Model with Growing Items

Based on the objective function (16) and constraint (19), the mathematical formulation of the EOQ inventory model with growing items is given by

$$\begin{aligned}&\operatorname{Min} TC = \left\{\tfrac{Dcw_0}{w_1} + \tfrac{Dr(w_1-w_0)^2}{2kw_1} + A\left(\tfrac{1}{T}\right) + \tfrac{hD}{2}(T)\right.\\ &\qquad\qquad \left. + \left(\tfrac{h+f}{2D}\right)\left(\tfrac{(S)^2}{T}\right) - h(S)\right\}\\ &s.t. \qquad T \geq T_{min}\\ &\qquad\qquad S \geq 0\\ &\qquad\qquad T > 0\end{aligned} \tag{20}$$

.

4 Solution Procedure

The objective function of the proposed problem (20) is convex (see "Appendix A"). On the other hand, the proposed mathematical model has a linear constraint, and consequently, this mathematical model is a convex continuous non-linear programming. Convex property states that if a feasible solution outperforms its neighbourhoods, i.e., local optima, then it is global optima as well. As a result, the optimum solution of the objective function (20) is calculated by employing partial derivatives. Calculating the partial derivative of the objective function (20) with respect to the cycle-length (T) and setting it equal to zero, the optimal solution of T is as follows:

$$\begin{aligned}\frac{\partial TC}{\partial T} &= \frac{-A - \left(\frac{h+f}{2D}\right)S^2}{T^2} + \frac{hD}{2}\\ &= 0 \to T = \sqrt{\frac{2DA + (h+f)S^2}{hD^2}}.\end{aligned} \tag{21}$$

Furthermore, calculating the partial derivative of objective function (20) with respect to the shortage quantity (S) and setting it equal to zero, the optimal solution of S is as follows:

$$\frac{\partial TC}{\partial S} = \left(\frac{h+f}{DT}\right) S - h = 0 \rightarrow S = \frac{hDT}{h+f}. \tag{22}$$

Substituting S from Eq. (22) in Eq. (21), the optimum cycle length is calculated as follows:

$$T = \sqrt{\frac{2DA}{hD^2\left(1 - \frac{h}{h+f}\right)}}. \tag{23}$$

Finally, based on the proposed concepts and formulas, solution steps of the optimization algorithm for the proposed EOQ inventory model with growing items are as follows:

STEP 1 Calculate $T_{\min}$ from Eq. (19).
STEP 2 If $T_{\min} \geq 0$ then problem is feasible and go to *STEP 3*, else problem is infeasible and go to *STEP 8*.
STEP 3 If $1 - h/h + f$ is positive, then problem is feasible and go to *STEP 4*, else problem is infeasible and go to *STEP 8*.
STEP 4 Calculate T from Eq. (23).
STEP 5 If $T \geq T_{\min}$, then $T^* = T$, else $T^* = T_{\min}$.
STEP 6 Calculate S^* using Eq. (22) considering T^*.
STEP 7 Calculate TC^* and y^* using objective function of (20) and Eq. (5), respectively, considering the obtained T^* and S^*, and report the optimal solution.
STEP 8 End

5 Numerical Example and Sensitivity Analysis

This section investigates a case study and its sensitivity analysis for the optimal solution. The example considers male broiler chickens raised in Lashgary poultry farm, located in Iran, Qazvin. To do so, we apply Richards growth function developed for this specific type of chickens. Based on Richards' growth curve, the growth curve of chickens is as follows (Goliomytis et al. [28]),

$$w_t = 6870\left(1 - 0.043e^{-0.036t}\right)^{1/0.0087}. \tag{24}$$

Figure 4 depicts daily basis weights of male broiler chickens calculated by Eq. (24).

The poultry company buys chickens when they are 2 days old and slaughter them at the age of 30 days. The reason why chickens are slaughtered at the age of 30 day lays in keenness of Iranian customers for buying chickens around 1200–1300 g weight. Thus, based on the Richard Growth function, the weight of a chicken at the beginning of the purchase is equal to 63 g ($w_0 = 63$g), and after 30 days, the weight reaches to 1266 g ($w_1 = 1266$g). Thus, Richard Growth function for this period can be obtained as in Fig. 5.

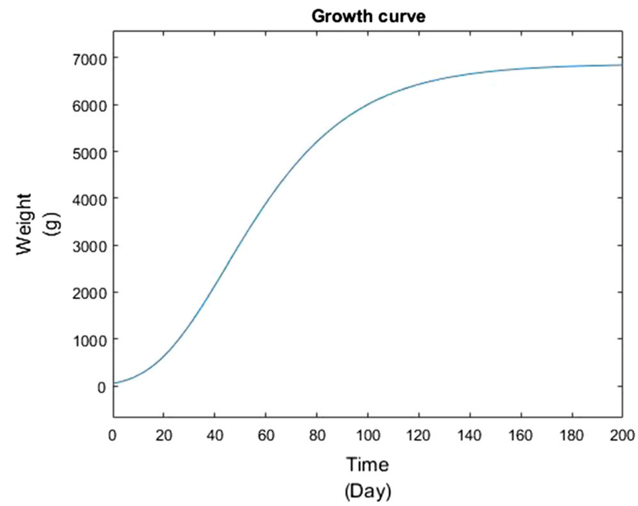

Fig. 4 Richards' growth curve

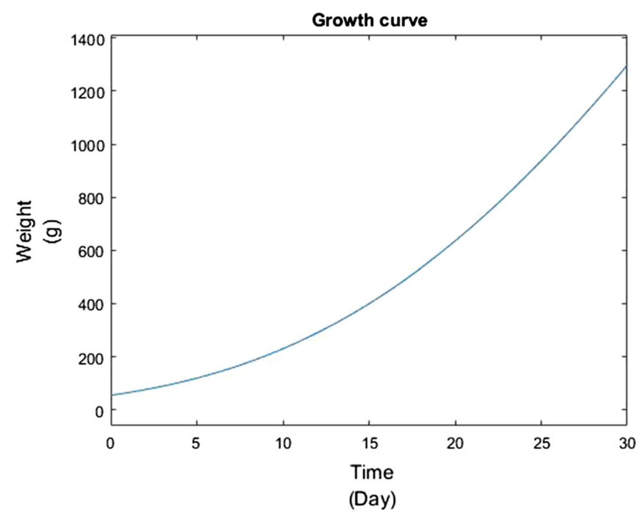

Fig. 5 Richards' growth curve for the real example

Since there is a limited curvature in the growth function in this interval, it can be approximated by a linear function using the minimum and maximum weight of the chickens. Using this approach, line slope would be 42 g per day, i.e., 15,330 g per year. Based on which, the approximate weight of each bought chicken is 84 g and its weight after 30 days would be around 1260 g. Due to the approximation, there is a small amount of discrepancy between our values and calculated values using Richard growth function. Therefore, the approximate growth function for this poultry company is shown in Fig. 6

Thus, we consider a real inventory system for the growing items with $k = 15,330$/chick/g/year, $D = 100,000$ g/year, $w_0 = 84$ gram, $w_1 = 1260$ gram, $ts = 0.01$ year, $c = \$0.3$/gram, $r = \$0.8$/gram/year, $h = \$0.4$/gram/year,

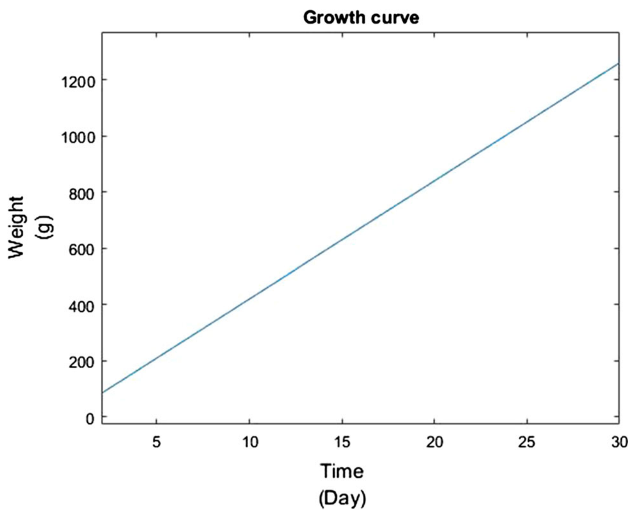

Fig. 6 Approximated growth curve for the real example

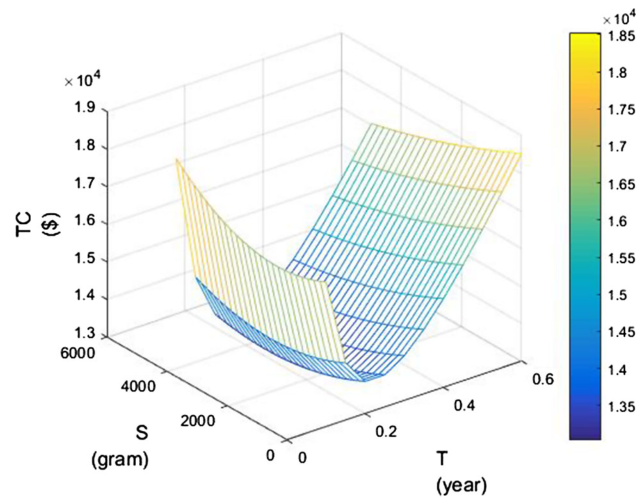

Fig. 7 Graph associated with TC according to S and T

$A = \$1000$/gram/year and $f = \$2$/gram/year. Due to the poultry company boundaries for information dissemination, the values for costs and other values are obtained using some vagueness and inaccuracy.

5.1 Example Optimization

This subsection shows the application steps of proposed optimization algorithm.

STEP 1 Calculate $T_{\min}$ using Eq. (19)

$$T_{\min} = 0.0867.$$

STEP 2 $T_{\min} > 0$ and the feasibility of problem is proven, go to *STEP 3*.

STEP 3 Since $1 - h/h + f = 0.8$, then the feasibility of problem is proven, go to *STEP 4*.

STEP 4 Calculate T using Eq. (23):

$$T = 0.2449.$$

STEP 5 $T > T_{\min}$, so $T^* = T = 0.2449$.

STEP 6 Calculate S^* using Eq. (22) considering T^*:

$$S^* = 4082.4.$$

STEP 7 Calculate TC^* and y^* using objective function of (20) and Eq. (5) respectively, considering the obtained T^* and S^*; and report the optimal solution:

$$TC^* = \$13028.8; \quad y^* = 19.4 \text{ chickens};$$
$$T^* = 0.2449 \text{ year; and } S^* = 4082.4 \text{ g}.$$

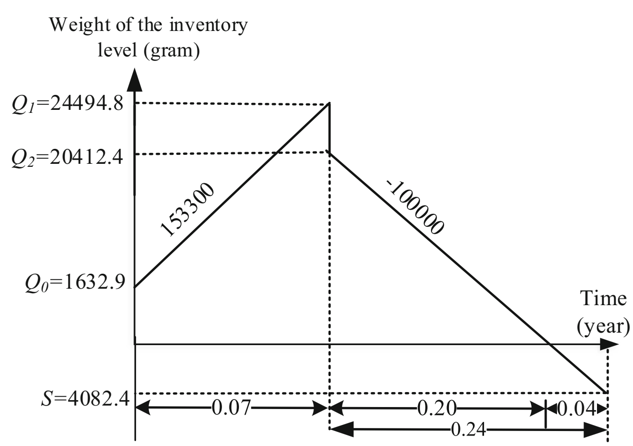

Fig. 8 Diagram of the weight of the inventory of real industrial case

STEP 8 End

Finally, the graph associated with the objective function according to the decision variables S and T is shown in Fig. 7. As it is depicted in Fig. 7, this function is convex. Moreover, Fig. 8 represents a schematic overview of the inventory graph associated with this example, wherein the duration of the breeding period is $t_1 = 0.0767$ year $= 28$ days, the consumption period is $t_2 = 0.2041$ year ≈ 75 days, and the shortage period is around $t_2 = 0.0408$ years ≈ 15 days. In addition, at the beginning of the cycle, chicks total weight is equal to $Q_0 = y^* w_0 = 1632.9$ g and their final weight after the breeding period is equal to $Q_1 = y^* w_1 = 24,494.8$ g.

5.2 Sensitivity Analysis

In this subsection, the impact of parameters on decision variables and the objective function is investigated. To do so, the

Table 2 Sensitivity analysis

Parameters	% in change	% in change		
		S^*	T^*	TC^*
k	− 30	0	0	9.4
	− 10	0	0	2.4
	10	0	0	− 1.94
	30	0	0	− 5.07
D	− 30	− 16.33	19.52	− 21.43
	− 10	− 5.13	5.40	− 6.94
	10	4.88	− 4.65	6.79
	30	14.01	− 12.29	19.98
w_0	− 30	0	0	− 3.65
	− 10	0	0	− 1.21
	10	0	0	1.22
	30	0	0	3.67
w_1	− 30	0	0	− 0.94
	− 10	0	0	− 0.80
	10	0	0	1.11
	30	0	0	4.00
A	− 30	− 16.33	− 16.33	− 10.23
	− 10	− 5.13	− 5.13	− 3.21
	10	4.88	4.88	3.05
	30	14.01	14.01	8.78
c	− 30	0	0	− 4.60
	− 10	0	0	− 1.53
	10	0	0	1.53
	30	0	0	4.60
r	− 30	0	0	− 6.59
	− 10	0	0	− 2.19
	10	0	0	2.19
	30	0	0	6.59
h	− 30	− 14.16	16.494	− 8.87
	− 10	− 4.33	4.52	− 2.71
	10	4.01	− 3.86	2.51
	30	11.26	− 10.12	7.06
f	− 30	38.01	3.50	− 2.12
	− 10	10.09	0.92	− 0.57
	10	− 8.39	− 0.76	0.48
	30	− 21.55	− 1.94	1.24

value of each parameter is changed by leaving other parameters unchanged. The amount of variation for each parameter and obtained results are given in Table 2.

Based on the information of Table 2, the following observations are made:

- Changes in shortage cost, holding cost, demand rate, and setup cost have a significant impact on the optimum amount of shortage. In addition, variation in purchasing cost, growth rate, approximated weight of new-born items, and approximated weight at the moment of slaughtering and feeding cost have no effect on optimum amount of shortage.
- Shortage cost change has an insignificant influence on optimum growth period length. Holding cost, demand rate, and setup cost changes have significant impacts on optimum growth period length. In addition, variation in purchasing cost, growth rate, approximated weight of new-born items, approximated weight at the moment of slaughtering, and feeding cost have no effect on optimum growth period length.
- Changes in approximated weight of new-born items, approximated weight at the moment of slaughtering, growth rate, purchasing cost, feeding cost, shortage cost, and holding cost have an insignificant influence on optimum objective function. Deviations in demand rate and setup have significant impacts on optimum objective function.

6 Conclusion and Future Research Directions

This research develops an EOQ inventory model for poultries with a case study. It is assumed that the growing rate of items could be estimated by a linear function. Then, using a real-world scenario, we assessed the applicability of this assumption in poultry industries. This EOQ inventory model studies a poultry company that fed new-born chickens until they reach to an ideal weight for consumers. The ideal weight is calculated based on the available data about the consumer demands of chicken. Furthermore, the managers aim to increase the system productivity by employing consumer tendency to wait for fresh food. To do so, shortage is permitted in the system in the form of fully back ordering. The main innovation of this lays in employing shortages for EOQ inventory models with growing items. In addition, in this study, a closed form procedure for solving such problems is proposed that helps managers to easily obtain the optimum policy.

The aim of the proposed EOQ inventory model is to determine the optimal values for shortage and cycle length that minimizes total inventory system costs including purchasing cost, setup cost, holding cost, food procurement cost, and shortage cost. To achieve these objective, a non-linear programming model for the inventory problem is obtained and the convexity of it is proven. Then, an exact algorithm for optimizing this inventory problem is developed. Finally, we studied a numerical example based on a real scenario with its sensitivity analysis to investigate the impact of parameters on optimum cycle length, the amount of shortage, and total cost. Finally, the following recommendations are suggested for future extensions of the EOQ inventory model with growing items:

- Considering animal mortality during the growth period in the form of product wastage.
- Addressing the perishability of items during consumption period.
- Storage capacity boundary can be considered in the growth period.
- Pricing policies for items can be taken into account.
- Considering availability of various suppliers and investigating procurement policies based on ordering items from a number of suppliers.
- Offering discount by suppliers.
- Considering different types of product in system.
- Developing the system under incentive policies such as permissible delays in payment and prepayment.

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Appendix A

This appendix proves the convexity of the objective function (20). The objective function of (21) is:

$$TC = \left\{ \frac{Dcw_0}{w_1} + \frac{Dr\,(w_1 - w_0)^2}{2kw_1} + A\left(\frac{1}{T}\right) + \frac{hD}{2}\,(T) + \left(\frac{h+f}{2D}\right)\left(\frac{(S)^2}{T}\right) - h\,(S) \right\}. \tag{A.1}$$

Therefore, the partial derivatives with respect to different decision variables are as follows:

$$\frac{\partial TC}{\partial T} = \frac{-A}{T^2} - \left(\frac{h+f}{2DT^2}\right)S^2 + \frac{hD}{2} \tag{A.2}$$

$$\frac{\partial^2 TC}{\partial^2 T} = \frac{2A}{T^3} + \left(\frac{h+f}{DT^3}\right)S^2 \tag{A.3}$$

$$\frac{\partial TC}{\partial S} = \left(\frac{h+f}{DT}\right)S - h \tag{A.4}$$

$$\frac{\partial^2 TC}{\partial^2 S} = \frac{h+f}{DT} \tag{A.5}$$

$$\frac{\partial^2 TC}{\partial T \partial S} = \frac{\partial^2 TC}{\partial S \partial T} = -\left(\frac{h+f}{DT^2}\right)S. \tag{A.6}$$

The Hessian matrix of the objective function is

$$\text{Hessian matrix} = \begin{bmatrix} \frac{\partial^2 TC}{\partial^2 T} & \frac{\partial^2 TC}{\partial T \partial S} \\ \frac{\partial^2 TC}{\partial S \partial T} & \frac{\partial^2 TC}{\partial^2 S} \end{bmatrix}. \tag{A.7}$$

Based on Hessian matrix, the quadratic form of objective function (20) is equal to

$$\text{Quadratic form} = \begin{bmatrix} T & S \end{bmatrix} \begin{bmatrix} \frac{2A}{T^3} + \left(\frac{h+f}{DT^3}\right)S^2 & -\left(\frac{h+f}{DT^2}\right)S \\ -\left(\frac{h+f}{DT^2}\right)S & \frac{h+f}{DT} \end{bmatrix} \begin{bmatrix} T \\ S \end{bmatrix} = \frac{2A}{T} \geq 0 \tag{A.8}$$

As it can be seen from (A.8), the quadratic form of the objective function is positive, and consequently, the objective function is convex.

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