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In this paper we propose an algorithm called Highly Optimised Tolerance (HOT) for solving a multi-stage, multi-product supply chain network design problem. HOT is based on power law and control theory. The proposed approach takes its traits from the local incremental algorithm (LIA), which was initially employed to maximise the design parameter (i.e. yield), particularly in the percolation model. The LIA is somewhat analogous to the evolution by natural selection schema. The proposed methodology explores a wide search space and is computationally viable. The HOT algorithm tries to make the system more robust at each step of the optimisation. The objective of this paper is to reduce the total cost of supply chain distribution by selecting the optimum number of facilities in the network. To examine the effectiveness of the HOT algorithm we compare the results with those obtained by applying simulated annealing on a supply chain network design problem with different problem sizes and the same data sets.

Keywords: optimisation; simulated annealing; supply chain design

1. Introduction

In today's competitive and global market, the success of an industry is reliant upon the management of its supply chains. Supply chain network design includes all the internal and external components of supply chain management. Nowadays, customers want the product at the minimum possible cost, so it is important that a firm organise the plants, retailer, supplier, distribution centres, and customer zone in such a manner that customers obtain the product at minimum possible cost and the firm should maintain a profit. In recent years, many researchers have proposed different types of models for supply chain network design. The model of supply chain network design changes from time to time due to customer demand. The supply chain network design problem is the most important decision-making problem that needs to be optimised for the long-term effective operation of the whole supply chain. An efficient supply chain network provides an optimal platform for effective supply chain management. Supply chain management is used to manage the cost, risk, profit margin, and inventory management and to collaborate with other factors that affect both the company and the customer. Information, products and funds flow constantly between the stages of the supply chain network. Distribution of the product is a key driver of the overall profitability of firms and directly influences supply chain cost.

The supply chain distribution model discussed here is related to the multi-product, multi-stage network design problem. The proposed problem is NP hard, and is not easy to solve using deterministic methods, therefore we use AI techniques to solve this type of problem. In this paper we propose a new optimisation approach based on the Highly Optimised Tolerance (HOT) algorithm proposed by Carlson and Doyle (1999). The HOT algorithm has a wide search space, and tries to make the system more robust at each stage of the optimisation. It is based on power law and control theory. The HOT algorithm provides a satisfactory result for the complex network design problem and is also computationally viable. Thus we used the HOT algorithm to solve the network design problem discussed here.

The remainder of the paper is organised as follows. Section 2 presents the relevant literature, which covers all aspects of supply chain network design, and also discusses the problem and method used in this paper. Section 3 covers the mathematical model of the multi-stage, multi-product supply chain network design. Section 4

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presents the solution methodology based on the HOT algorithm. Section 5 makes a comparison with simulated annealing. Section 6 describes the simulated annealing methodology, and Section 7 discusses the experimental analysis of the proposed problem. Section 8 reports the computational results and Section 9 concludes.

2. Relevant literature

Supply chain network design provides an optimal platform for efficient supply chain management. It colligates various facilities such as suppliers, plants, distribution centres, retailers, warehouses and customer zones such that customers receive the benefit of the product at the minimum possible cost and the firm maintains their profit margin. The supply chain is a network of suppliers, manufactures, distributors and customers interconnected by transportation, information sharing, and financial infrastructure (Chopra 2003). Suppliers are at the start of the network and provide raw material to the plant. Every plant has more than one supplier. The plant provides the manufactured product to the distribution centres. Each plant and distribution centre has a certain capacity and they cannot exceed that capacity. Customers are at the end of the supply chain; usually, each customer is assigned to a single distribution centre. In recent years, the supply chain design problem has gained importance due to the increasing competitiveness introduced by market globalisation (Thomas and Griffin 1996). Firms are demanding a better customer service level while they are forced to minimise cost and maintain the profit margin. The supply chain comprises different echelons, such as suppliers, plants, warehouses, retailer, distribution centre, and customers. Surana *et al.* (2005) proposed a supply chain network from the perspective of a complex adaptive system, and discuss the various aspects of a complex supply chain.

Conventionally, marketing, distribution, planning, manufacturing and purchasing organisations along the supply chain have operated independently. Although these firms have their own goals, there is a need for a mechanism that can produce collaboration among these layers. Supply chain design is a mechanism by which all the firms are colligates, while their original objectives remain the same. In a supply chain, the flow of goods between a supplier and the customer passes through several stages, and each stage may consist of many facilities (Sabri and Beamon 2000). Supply chain network design (SCN) is used to provide an effective and expedient site for supply chain management (SCM). Yang *et al.* (2002) proposed a system for supply chain network management which consists of an optimised supply chain network design model, an integrated planning module for distribution operations covering raw material supplier to customers, a model management supporting flexible mathematical modelling and a data management. Hongwei *et al.* (2009) developed a simulation-based optimisations for a stochastic multi objective production–distribution network design. A multi-product, multi-stage, and multi-period scheduling model is proposed to deal with multiple incommensurable goals for a multi-echelon supply chain network with uncertain market demands and product prices by Chen and Lee (2004). Park (2005) proposed an integrated approach for production and distribution planning in supply chain management. A review on the integration of production and distribution planning in supply chain is carried out by Erenguc *et al.* (1999).

In an effective supply chain network design, it is necessary to choose the facilities (plants and distribution centres (DCs)) to be opened and for the network design to satisfy customer demand at minimum cost and for the firm to obtain maximum profit. It is a facility location allocation problem. Supply chain network design is one the most important mechanisms used for long-term efficient supply chain management. The SCN determines the number of suppliers, the location, types of plant, capacity, types of warehouses and types of distribution centres to be used. The SCN also determines the distribution channels and the amount of material and items to consume, produce, and ship from supplier to customers. In the network design problem, a plant receives material from several suppliers and then supplies several distribution centers (DCs), which finally supply to the customer.

Hence the multi-stage supply chain network (MSCN) is used to describe the supply chain network. When the facilities (plants and DCs) have a certain capacity, the problem is a capacitated location problem. Since the multi-stage problem is difficult to solve optimally, particularly if capacity constraints are imposed on both plant and DCs, researchers have utilised many heuristic approaches to solve the supply chain network design problem. Riccardo *et al.* (2008) developed an optimisation model for the dynamic facility location allocation problem.

Many researchers have proposed various types of heuristic approaches for solving the supply chain network design problem under various circumstances. Jayaraman and Pirkul (2001) proposed a heuristic approach based on Lagrangian relaxation for the single-source, multi-product, multi-stage SCN design problem. Altiparmak *et al.* (2009) proposed a steady-state genetic algorithm approach for the multi-product supply chain network design problem. Another heuristic approach based on a genetic algorithm for the supply chain network design problem was proposed by Altiparmak *et al.* (2005). Jawahar and Balaji (2007) also developed a heuristic approach based on a genetic algorithm for the two-stage supply chain distribution problem associated with a fixed charge. Yeh (2006) proposed an efficient memetic algorithm for the multi-stage supply chain network problem. Jayaraman and Ross (2003) developed a simulated annealing methodology for distribution network design and management. Syarif *et al.* (2002) developed a spanning-tree-based GA approach for the multi-source, single-product, multi-stage SCN design problem. Altimarpark *et al.* (2006) proposed a new solution procedure based on genetic algorithms to find the set of Pareto-optimal solutions for multi-objective SCN design problems. Gen and Syarif (2005) developed a hybrid genetic algorithm approach for multi-time-period production/distribution planning. Michalewicz *et al.* (1991) proposed a non-standard genetic algorithm used to solve the fixed charge transportation problem. Hang and Park (2002) presented a combined model of network design and production/distribution planning for a SCN. Chan *et al.* (2005) proposed a hybrid approach of a genetic algorithm and an AHP (Analytic Hierarch Process) for the production and distribution problem in multi-factory supply chain models. Beamon (1998) proposed models and methods for supply chain design and analysis. Jang *et al.* (2002) proposed a combined model of network design and production/distribution planning for a supply chain network. A multi-objective approach for simultaneous strategic and operational planning in supply chain design was developed by Sabri and Beamon (2000a). Aytug *et al.* (2003) used a genetic algorithm to solve the production and operations management problem. Zhou *et al.* (2002) proposed a genetic algorithm approach for the balanced allocation of customers to multiple distribution centres in the supply chain network. Kannan *et al.* (2009) proposed a genetic algorithm and particle swarm optimisation for the closed-loop supply chain problem.

In the last few decades there have been various types of AI techniques used to optimise the supply chain network design problem. Reeves (1995b) proposed different AI techniques for modern heuristic techniques for combinatorial problems. In this paper, we propose a heuristic approach based on HOT for a single-source, multi-product, multi-stage supply chain network design problem. HOT is a new framework for studying the behaviour of complex systems in an uncertain environment. It was developed by Carlson and Doyle (1999), taking inspiration from the behaviour of biological organisms and advanced engineering technologies. HOT is based on power law and control theory. Carlson and Doyle (2000a) also proposed a HOT approach for robustness and design in complex systems. Carlson and Zhou (2000) propose that HOT is a mechanism for power laws in complex systems based on the robust design of systems in uncertain environments. Xiangning and Zhiquin (2008) developed an HOT model to describe the statistics of robust complex in uncertain environment and used to analyse the electric blackouts which occurred in the North America Power grid. Wojcik (2007) developed a HOT-inspired model for large-scale systems. They showed how power laws in event size emerge from minimisation of the expected cost in the face of design trade-offs. HOT is a mechanism for complexity, in which non-generic features emerge without being introduced directly. HOT leads to power laws, and, more importantly, to systems that are robust to common and fragile design flaws and rare perturbations. The main feature of the HOT state is the sensitivity to unexpected perturbations or systematic changes in the environment. Robert *et al.* (2001) investigate the consequences of two new features in highly optimised tolerance (HOT), a mechanism which describes how complexity arises in systems which are optimised for robust performance in the presence of a harsh external environment. Initially, HOT was used in the context of a variety of specific applications, including the Internet, the electric power grid, wildfires and biological networks. Lin and Bo (2008) used HOT for the analysis of blackout data from North American power grids. HOT is suitable for both local and global optimisation.

3. Mathematical model

The mathematical model discussed in this section is a mixed-integer program for the single-source, multi-product, multi-stage SCN design problem. In this problem we take a subset of plants and DCs to be opened and design the distribution network strategy that will satisfy all capacity and customer demand for all

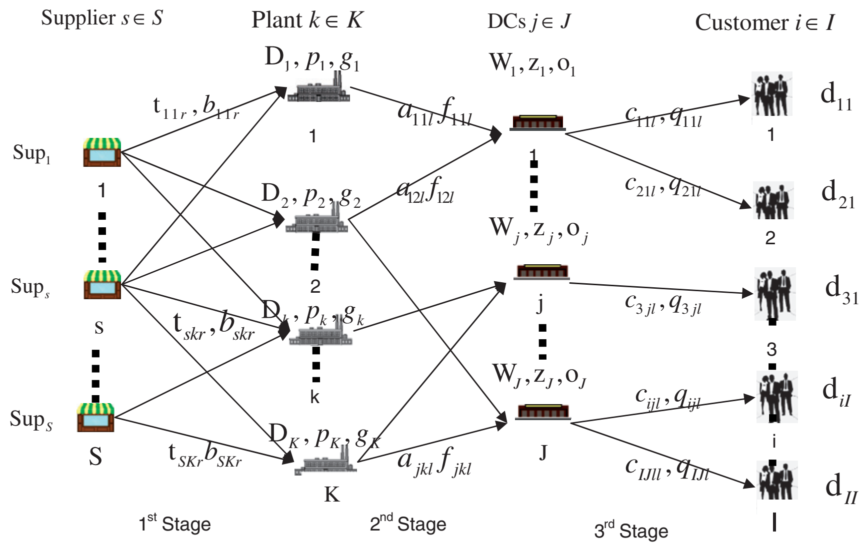

Figure 1. A simple three-stage supply chain network adopted from Altiparmak (2009).

products bought by customers with minimum cost. In this probem we make certain assumptions. The first is that the number of suppliers and their demand (capacity) are known. Second, the number of plants, the number of DCs and their maximum capacity are known. Figure 1 shows the multi-stage, multi-product supply chain network design problem.

The following notation is used to define the mathematical model of the supply chain network design problem.

Indices

- I set of customers $(i \in I)$
- J set of DCs $(j \in J)$
- K set of plants $(k \in K)$
- L set of products $(l \in L)$
- S set of suppliers $(s \in S)$
- R set of raw materials $(r \in R)$

Variables

- x_j 1 if DC j is opened, 0 otherwise
- p_k 1 if plant k is opened, 0 otherwise
- y_{ij} 1 if DC j serves customer I, 0 otherwise
- b_{skr} quantity of raw material r shipped from supplier s to plant k
- z_{lk} quantity of product l produced at plant k
- q_{ijl} quantity of product l shipped from DCj to customer i
- f_{jkl} quantity of product l shipped from plant k to DCj

Parameters

- Q_k capacity of plant k
- T_j annual throughput at DC j
- Q_{sr} capacity of supplier s for raw material r
- D_{il} demand for product l at customer i
- N maximum number of DCs
- P maximum number of plants
- F_j annual fixed cost for operating DCj
- C_k annual fixed cost for operating plant k
- U_j unit cost of throughput for a DC at site j
- U_{lk} unit production cost for product l at plant k
- c_{ijl} unit transportation cost for product l from DC j to customer i
- a_{jkl} unit transportation cost for product l from plant k to DCj
- P_{skr} unit transportation and purchasing cost for raw material r from supplier s to plant k
- s space requirement rate of product l in a DC
- m_1 capacity utilisation rate per unit of product l
- u_{rl} utilisation rate of raw material r per unit

The problem can be formulated as follows:

$$\min Z = \sum_j F_j z_j + \sum_i \sum_j \sum_l U_j D_{il} y_{ij} + \sum_k C_k p_k + \sum_l \sum_k U_{lk} x_{lk} + \sum_s \sum_k \sum_l P_{skr} b_{skr} + \sum_i \sum_j \sum_l c_{ijl} q_{ijl} + \sum_j \sum_k \sum_l a_{jkl} f_{jkl}, \tag{1}$$

subject to

$$\sum_j y_{ij} = 1, \quad \forall i, \tag{2}$$

$$\sum_i \sum_l s D_{il} y_{ij} \le T_j z_j, \quad \forall j, \tag{3}$$

$$\sum_j z_j \le N, \tag{4}$$

$$q_{ijl} = D_{il} y_{ij}, \quad \forall i, j, l, \tag{5}$$

$$\sum_k f_{jkl} = \sum_i q_{ijl}, \quad \forall j, l, \tag{6}$$

$$\sum_k b_{skr} \le Q_{sr}, \quad \forall s, r, \tag{7}$$

$$\sum_k u_{rl} x_{lk} \le \sum_s b_{skr}, \quad \forall r, k, \tag{8}$$

$$\sum_l m_l x_{lk} \le Q_k p_k, \quad \forall k, \tag{9}$$

$$\sum_j f_{jkl} \leq x_{lk}, \quad \forall k, l, \tag{10}$$

$$\sum_k p_k \leq P, \tag{11}$$

$$z_j = \{0, 1\}, \quad \forall j, \tag{12}$$

$$p_k = \{0, 1\}, \quad \forall k, \tag{13}$$

$$y_{ij} = \{0, 1\}, \quad \forall i, j, \tag{14}$$

$$b_{skr} \geq 0, \quad \forall s, k, r, \tag{15}$$

$$x_{lk} \geq 0, \quad \forall l, k, \tag{16}$$

$$q_{ijl} \geq 0, \quad \forall i, j, l. \tag{17}$$

In the proposed mathematical model the objective function is to minimise the total cost of the supply chain for the single-source, multi-product, multi-stage SCN design problem. This consists of the annual fixed cost of operating the plants and DCs, the annual throughput cost for a DC, the variable cost of production and distribution, and the transportation costs of raw material from the supplier to the plant, the finished product from the plant to the DCs and from the DCs to customers. Constraint (1) shows the unique assignment of a DC to a customer. Constraint (2) shows the capacity constraint for DCs. Constraint (3) limits the number of DCs that can be opened. Constraints (4) and (5) are used for the satisfaction of customer and DC demand for all products. Constraint (6) is used for the raw material supply restriction. Constraint (7) shows the raw material requirement for production. Constraint (8) is the capacity constraint for a plant. Constraint (9) gives the limitation on the total quantity shipped from the manufacturing plant to customers through DCs, which cannot exceed the production quantity produced by the plant. Constraint (10) shows the limitation on the number of plants that can be opened. Constraints (11)–(13) restrict the decision variables z_k, p_k, and y_{ij}. Constraints (14)–(16) impose non-negativity restrictions on decision variables b_{skr}, x_{lk}, and q_{ijl}. The capacitated plant location problem is NP hard and this problem integrates the capacitated plant location, distribution and production problem so that the single-source multi-product, multi-stage supply chain network design problem is also NP hard.

4. Proposed optimisation methodology: Highly optimised tolerance inspired procedure

Highly optimised tolerance (HOT) is a mechanism for power laws that coalesces the perspective of statistical physics with engineering methods for building up a robust, highly interconnected system. HOT is fundamentally based on the control theory and power law that is used in many problems such as the forest fire management model, the percolation model, the sand pile model, biological cell survival systems and Internet file transmission. Due to the various features of HOT we used it as an optimisation technique. In recent years, many researchers have used HOT in various circumstances to solve various design problems.

4.1 *Motivation*

In complex systems, it is often observed that the size of the triggering event is independent of the size of the initiating event (Carlson and Doyle 1999). In HOT systems, we observe how power laws in the event size emerge from minimisation of the cost in the face of design trade-off. HOT is a new framework for understanding certain aspects of the complexity of designed or engineered systems. Due to the behaviour of HOT we used the highly optimised tolerance inspired procedure as an optimisation method. The problem discussed in this paper is the multi-product, multi-stage supply chain distribution problem. This problem is very complex due to the capacity constraint and excessive variables, and is difficult to solve using deterministic methods. Therefore, we use different heuristic approaches to solve this type of NP-hard, mixed-integer problem. Many algorithms converge at local minima and do not give satisfactory results, whereas the HOT algorithm has a better search space and does not converge at local minima. Therefore, in this paper we use HOT to minimise the total supply chain distribution cost.

4.2 *Background information*

Carl *et al*. (2001) and Carlson and Doyle (1999, 2000a) proposed a mechanism, inspired by biological organisms and advanced engineering technology, to study the behaviour of complex systems as well as the consequence of their design. They referred to this mechanism as Highly Optimised Tolerance (HOT). It is based on control theory and a power law that has been applied to deal with several problems pertaining to forest fire management, the forest fire percolation model, the biological cell survival system and the Internet file transmission traffic problem, among others (Moritz *et al*. 2005). It has been assumed that power laws are ubiquities in natural as well as in artificial systems. A literature survey by Bak (1996) has revealed that few models (such as forest fire percolation and sand piles) have been studied in terms of Self-Organised Critically (SOC) or Edge of Chaos (EOC). The application of deliberate design or evolution by natural selection in a system is the key idea concerning HOT that is applied to tune the system and leads towards a highly structured and efficient operating state of high yield value (objective value). Local external disturbances, for example a cascading failure event, also occur in the system. This failure reduces the number of particles in the connected cluster model in certain areas of the system. The regions affected due to failure events and the occurrence of external perturbations are given by a specific relation. These specific relations when represented in mathematical form are known as power laws. Power laws are common characteristics of various complex interconnected systems. Power outage frequency, the forest fire phenomenon, the biological cell survival system, and the Internet file transmission traffic problem are cited examples that follow power law distributions and are used to illustrate the HOT concept. Of these, two models that are frequently used by Carlson and Doyle (1999) are forest fires and percolation.

Initially, to define basic concepts, the HOT framework was applied to forest fire management. It is described as the optimisation of barrier patterns around the most sensitive areas so that the region burned due to random events (fire) can be minimised and consequently wood yield (objective value) is maximised. Further, consider that a spark is dropped accidently in a random system that has a density equivalent to the designed system. Here two cases arise: if the spark hits a vacant site, nothing burns; however, in the other case, trees within the connected cluster are burned. It is observed in the literature that the probability of occurrence of large events is smaller than that of small events (Carlson and Doyle 1999).

In essence, HOT is a mechanism that helps the designed system exhibit high performance despite an uncertain environment. A highly structured, non-generic, self-dissimilar internal configuration and a robust, yet fragile, external behaviour system can be the outcome of a small amount of design for a sophisticated system. This design is carried out by leading the initial step towards complicated structures. To analyse the basic dissimilarity between the random and the designed system, an alternative mechanism (HOT) for the power law has been proposed that results in the following characteristics of the designed system.

- High efficiency, performance, and robustness to uncertainties.
- Hypersensitivity to design flaws and unanticipated uncertainties.
- Non-generic, specialised, and structured configuration.
- Power law.

The optimisation of an objective function with respect to certain constraints results in the above attributes of the designed system. To optimise the design criterion (yield) in the percolation model, Carlson and Doyle (1999) described a local incremental algorithm, which is the subject of the following subsection.

4.3. Power law

A power law is a special kind of mathematical relationship between two quantities. According to a power law, a non-negative random variable X having probability distribution P_r is said to have a power law distribution if

$$P_r[X \geq x] \sim cx^{-\beta}. \quad (18)$$

For constant $c > 0$ and $\beta > 0$, the power law distribution depends upon the power (β) and varies according to the power value β. The power law is the key point of the HOT optimisation techniques. Clauset *et al*. (2009) proposed a

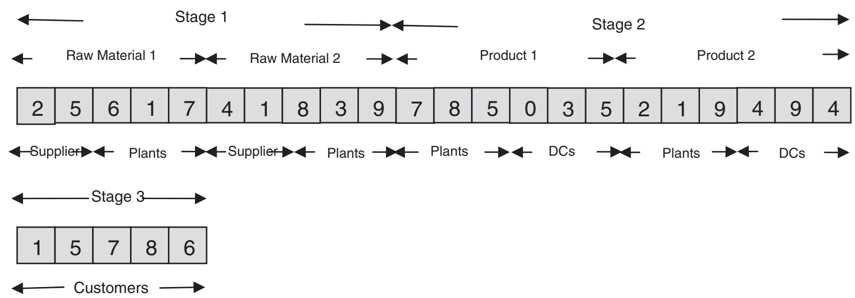

Figure 2. Representation of the initial population of a multi-product, multi-stage supply chain network.

power law distribution for imperial data. When the probability of measuring a particular value of some quantity depends upon the power of that value, then the quantity exhibits power law behaviour.

4.4. Evolution to the HOT state (local incremental algorithm)

In general, global optimisation is not always sufficiently applicable to introduce design to biological and engineering systems. These systems acquire a detailed state by searching for improved local alterations in the configuration after each step. The phenotype is utilised in a biological system, which differs conceptually from the genotype. In an engineering system, HOT states are clearly distinct, as specific designs are free from any occurrences. A design parameter such as density is assumed to be the phenotype and the obtained configuration is the genotype. Thus the genotype can evolve to the phenotype by applying the natural selection schema to the design parameter in the above two cases.

In addition, a system can be simplified to attain a specific state if a design parameter such as density is optimised using an evolutionary algorithm. This evolution involves a large number of continuous configurations of the system corresponding to a particular yield at any stage. In this state, the system follows a power law distribution. A probability distribution and a constraint on the optimisation are the two basic ingredients of the HOT state. The implementation procedure in the context of the supply chain network design problem is presented in the following subsection.

4.5. HOT-inspired mathematical model

The proposed HOT-inspired model is based on the power law. According to the HOT model the selection procedure depends upon the power law equation. In this model we assume that the probability of minimising cost depends upon the initial population. Let the population of the decision variable be X_k, with an initial population x_k and probability P_k. The initial population of the supply chain network design problem depends upon the parameter R_k. According to the power law,

$$x_k = (R_k)^{\beta}. \tag{19}$$

For some fixed β we want to minimise the total supply chain cost. If the total supply chain cost is

$$Z = \sum_{k=0}^{pop\_size} P_k x_k, \tag{20}$$

subject to the constraint

$$\sum_{k=0}^{pop\_size} R_k = N, \tag{21}$$

where N is a constant, from Equations (19), (20) and (21), using the Lagrangian multiplier, we obtain

$$P_k = N(x_k)^{-(1-(1/\beta))}, \tag{22}$$

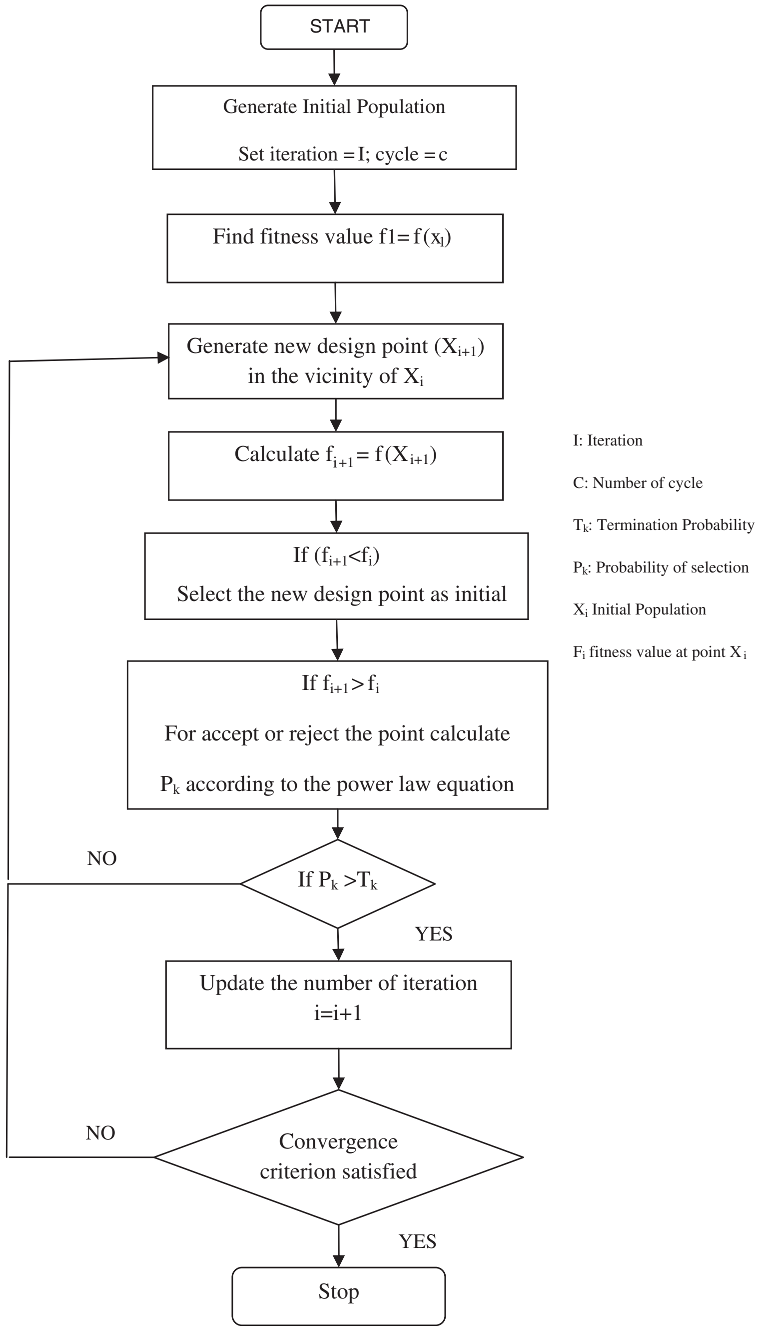

Figure 3. Block diagram of the proposed HOT approach.

$$P_k \propto (x_k)^{-(1-(1/\beta))}. \tag{23}$$

From Equation (23) we see that the probability of selection depends upon the initial population. We use this equation as a power law equation. The initial population of the problem is shown in Figure 2. Suppose that the fitness value of the initial population is f_k, then from Equation (23)

$$P_k \propto (f_k)^{-(1-(1/\beta))}. \tag{24}$$

The equation for the probability of selection is calculated from Equation (24). This is called the power law equation of the proposed mathematical model.

4.6 *Minimum selection probability*

In the proposed approach, T_p is assigned to select any point from the initial population. It plays a key role in reducing the total cost of the supply chain network, and thus makes the proposed approach computationally economical. As T_p is increased, the computational time is reduced; however, the resulting value of the cost becomes poorer. For example, if T_p is increased from 0.5 to 0.6, the computational time is reduced by 10%, but the final solution is worse. Moreover, to reduce the supply chain cost, after each iteration P_R of the supply chain is also increased. The main idea of the selection probability is to calculate a better probability of the cost function to make the system more reliable.

4.7 *Stopping criterion*

The selection of the stopping criterion is also an important factor for an efficient procedure that should be applied to stop the procedure after appropriate function evaluation. The following are the two stopping criteria used in the proposed procedure.

- Once the number of iterations attains the pre-assigned maximum value, the stopping criterion is achieved.
- Another criterion is in terms of a fixed value of the cost. When the cost value does not improve in a certain number of iterations, this indicates that there is a very small possibility of obtaining further improvement of the result and thus the stopping criterion is achieved.

4.8 *Block diagram of highly optimised tolerance*

The block diagram of Figure 3 shows the proposed steps involved in the highly optimised tolerance (HOT) approach.

5. Comparison with simulated annealing

To examine the effectiveness of the proposed highly optimised tolerance we use simulated annealing. Simulated annealing is a random search technique developed by Kirkpatrick *et al*. (1983). Jayaraman and Pirkul (2001) proposed simulated annealing for the distribution network design and management problem. Section 6 discusses the solution procedure of the simulated annealing approach. We compare the results obtained using the proposed HOT methodology and simulated annealing using the same data sets. Comparison of these two algorithms gives different values for the fitness function for the same data sets and, on the basis of the fitness value and convergence graph, we demonstrate the effectiveness of HOT.

6. Simulated annealing

The simulated annealing method is based on simulation of the thermal annealing of a critically heated solid. It is a probabilistic method proposed by Kirkpatrick *et al*. (1983) for calculating the global minima of a cost function. Jayaraman and Pirkul (2001) used the simulated annealing methodology for the distribution network design problem. The methodology is used in many areas of engineering. Starting from an initial solution, simulated

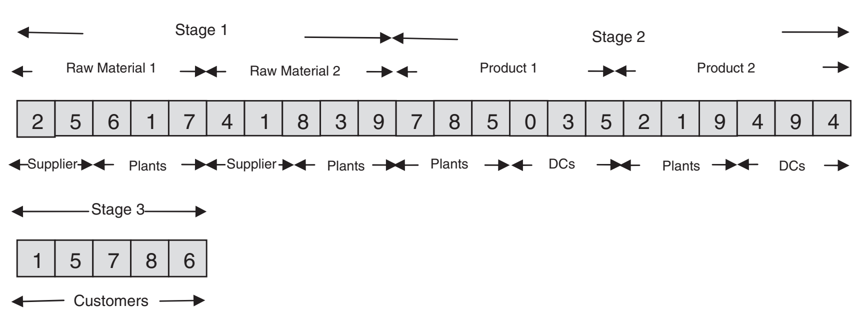

Figure 4. Representation of the initial population of the multi-product, multi-stage supply chain network.

annealing generates a new solution in the vicinity of the initial solution. We calculate the change in the objective value $\Delta f = f_i - f_{i+1}$. For the minimisation problem, if the change in the objective function is negative, then we take the new point as the initial point. If Δ is positive, then we use the metropolis criterion in which, accepting the design point, there is a specified probability denoted by $\exp[-(\Delta f/T)]$, where T is the parameter used as the initial temperature. In simulated annealing the quality of the final solution is not affected by the initial guess (see Figure 4).

Table 1. Unit transportation and purchasing cost for raw material 2 from supplier to plant *k*.

	Plant			
Supplier	1	2	3	4
1	5	4	2	7
2	6	8	4	6

Table 2. Unit transportation and purchasing cost for raw material 1 from supplier to plant *k*.

	Plant			
Supplier	1	2	3	4
1	5	4	2	7
2	6	8	4	6

Table 3. Unit transportation cost for product 1 from plant *k* to DC*j*.

	Distribution centre			
Plant	1	2	3	4
1	3	5	8	5
2	5	8	6	3
3	6	9	3	6
4	7	4	6	9

Table 4. Unit transportation cost for product 2 from plant *k* to DC*j*.

	Distribution centre			
Plant	1	2	3	4
1	3	5	8	5
2	5	8	6	3
3	6	9	3	6
4	7	4	6	9

Table 5. Unit transportation cost for product 1 from DC*j* to customer *i*.

	Customer			
Distribution centre	1	2	3	4
1	5	6	7	5
2	6	9	4	9
3	4	6	8	5
4	4	5	8	3

Table 6. Unit transportation cost for product 2 from DC*j* to customer *i*.

	Customer			
Distribution centre	1	2	3	4
1	5	6	7	5
2	6	9	4	9
3	4	6	8	5
4	4	5	8	3

Table 7. Demand of product *k* by customer *i*.

	Product	
Customer	1	2
1	800	1200
2	1200	2100
3	500	700
4	700	650

Table 8. Production cost of product l at plant *k*.

	Product	
Plant	1	2
C1	100	400
C2	120	340
C3	90	450
C4	100	325

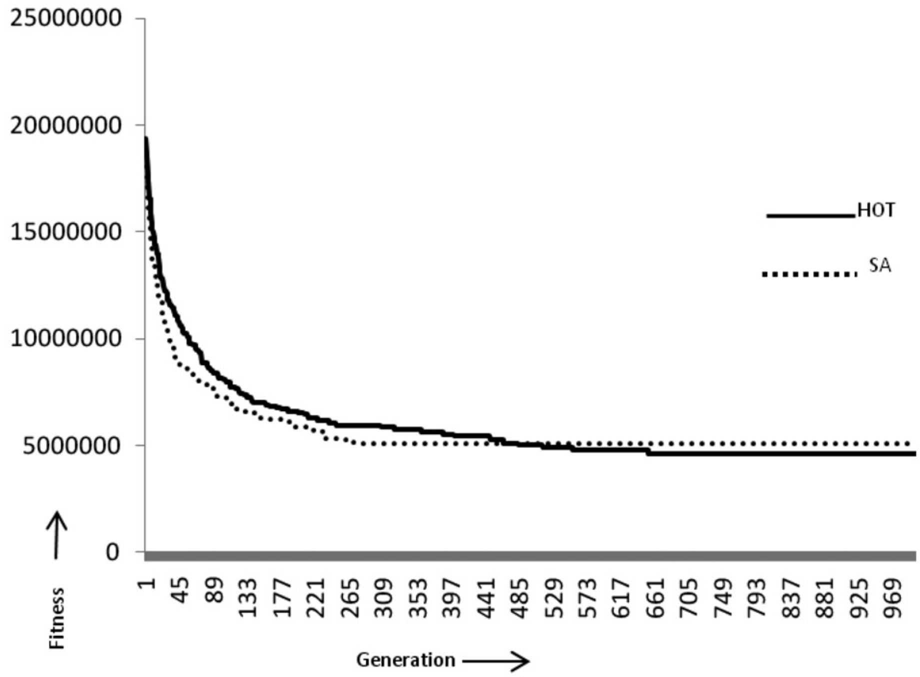

Figure 5. Convergence graph of the proposed techniques studied.

Table 9. Raw material 1 supplied from the supplier to the plant.

	Plant			
Supplier	P1	P2	P3	P4
S1	2776		846	783
S2	890		3897	1858

Table 10. Raw material 2 supplied from the supplier to the plant.

	Plant			
Supplier	P1	P2	P3	P4
S1	458		6999	3854
S2	5591		1599	399

6.1 *Pseudo-code of simulated annealing*

Simulated annealing

Generate the initial population X_i
Set the initial parameters (n = number of iterations, T = initial temperature, c = temperature reduction factor, E = number of cycles)
Repeat (until termination)
While (iter < iter_max) {
Calculate fitness value $f_i = f(X_{i+1})$
Set iteration $i = 1$; cycle $p = 1$
Generate new population X_{i+1} in the vicinity of X_i
For accept X_{i+1} or reject using the metropolis criterion
Update iteration number $i = i + 1$
Is number of iteration $i = n$
Update the number of cycles as $p = p + 1$
Iter++
Reduce temperature
}

Table 11. Product 1 supplied from the plant to the DC.

	Distribution centre			
Plant	D1	D2	D3	D4
P1		600	683	
P2				
P3		834	54	
P4		928	101	

Table 12. Product 2 supplied from the plant to the DC.

	Distribution centre			
Plant	D1	D2	D3	D4
P1		425	675	
P2				
P3		434	2533	
P4		355	228	

Table 13. Product 1 supplied from the DC to customers.

	Customer			
DC	C1	C2	C3	C4
DC1				
DC2	800	1200	500	
DC3				3000
DC4				

Table 14. Product 2 supplied from the DC to customers.

	Customer			
DC	C1	C2	C3	C4
DC1				
DC2		2100	750	
DC3	1200			600
DC4				

Table 15. Optimal values obtained by running different algorithms.

Problem instance	HOT approach	SA
1	13,525,515	14,521,169
2	13,528,930	14,347,784
3	13,228,319	14,102,391
4	13,223,479	14,648,357
5	12,863,627	13,364,762
6	28,944,054	30,096,738
7	30,120,072	31,467,552
8	30,428,257	31,266,597
9	51,706,754	50,327,932
10	52,593,739	54,273,483
Average	26,016,275	26,841,677

Table 16. Optimal values obtained at different selection probabilities.

Selection probability	Fitness value
0.995	3,999,226
0.975	4,492,860
0.875	4,674,086
0.865	4,710,251
0.855	4,752,528

7. Experimental study

The proposed problem is a multi-stage, multi-product supply chain network design problem. We have considered different problem sets and have used different heuristic approaches to solve the proposed problem. The data sets that were randomly generated according to the problem are shown in Tables 1–8. The parameter settings of HOT and the SA are as follows. The termination probability T_k used in the highly optimised tolerance approach is taken as 0.855 and the value of the power β is taken as 0.8. In simulated annealing the temperature reduction factor is taken as 0.6. Tables 1 and 2 show the transportation cost for raw materials from the suppliers to the plants, and all costs are unit cost per unit product. Tables 3 and 4 show the transportation costs for products from the plants to the DCs. Tables 5 and 6 show the transportation costs for products from the DCs to customers. Table 7 shows the demand of product by customers. Table 8 shows the production cost of the product at the plant. Using the data sets we calculated the different optimal values for different data sets. For a reasonable comparison between these approaches we take an initial population of about 500 and the number of generations is about 1000 for all approaches.

8. Computational result and discussion

The Highly Optimised Tolerance (HOT) gives better results than simulated annealing. The convergence graph shows that HOT has a better search space than other heuristic approaches (Figure 5). It does not converge at local minima. Tables 9–14 show the results obtained. Tables 9 and 10 show the quantity of raw material supplied from the supplier to the plants. Tables 11 and 12 show the quantity of product supplied from the plants to DCs. Tables 13 and 14 show the quantity of product supplied from DCs to customers. Table 15 shows the fitness values obtained using the proposed approach and simulated annealing for different configurations and data sets. We see that the fitness value obtained by HOT is smaller than that for the simulated annealing approach. To check the effect of selection probability on the fitness value we calculate the fitness value at different selection probabilities. Table 16 shows the fitness values obtained for different selection probabilities and we see that an increase in selection probability is accompanied by a decrease in the fitness value. The flow of product from suppliers to customers shows that only three plants (P_1, P_3, P_4) and two DCs (DC_2, DC_3) are open at a time. In this problem we assumed that customers are assigned to single DCs, which is clearly shown by Tables 13 and 14.

9. Conclusion

In this paper we have proposed a new optimisation technique called highly optimised tolerance (HOT) for the multi-stage, multi-product supply chain network design problem. The objective of the problem is to minimise the total supply chain distribution cost. HOT is based on power law and control theory. To investigate the effectiveness of the proposed algorithm we compared the result with those obtained using simulated annealing (SA) on the same data sets. We find that the proposed algorithm gives better results than SA. The convergence graph (Figure 5) shows that HOT does not converge at local minima, and also has a better search space than SA. HOT depends upon the power of the power law equation, denoted by β, and also the selection probability. An increase in the selection probability gives the minimum fitness value and less computational time. Furthermore, HOT can be modified to take into account more realistic aspects of the problem such as a dynamic environment and stochastic demand. In the future we plan to use new approaches based on tabu search or a memetic algorithm for the multi-product supply chain distribution problem.

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