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Fuzzy multi-criteria decision making model for different scenarios of electrical power generation in Egypt

Hesham A. Hefny [a], Hamed M. Elsayed [b,*], Hisham F. Aly [c]

[a] *Institute of Statistical Studies and Research, Cairo, Egypt*
[b] *Central Agency for Public Mobilization and Statistics, Cairo, Egypt*
[c] *Atomic Energy Authority, Cairo, Egypt*

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Abstract In the analytic network process (ANP) a hierarchy or network is created to represent a decision and establishes a matrix containing the pair wise comparison judgments for the elements linked under a parent element. A priority vector of relative weights for these elements is derived. Then all the priority vectors are appropriately weighted and summed to obtain the overall priorities for the alternatives of a decision. In this paper we will develop an efficient fuzzy ANP model which helps decision makers to choose among the alternatives for the Egyptian scenarios of electrical power generation.

1. Introduction

Fuzzy rule-based systems have been widely used in a variety of engineering areas such as data mining, pattern recognition, system identification, and process control [1]. Fuzzy logic is a key tool to express knowledge of domain experts so that valuable experience of human beings can be incorporated into controllers design and applied to handle real-life situations that the classical control approach finds difficult or impossible to tackle [2].

The analytic network process (ANP) is used for tackling multi-attribute decision-making problems in real situations when there is interrelation among decision criteria or alternatives. In the traditional formulation of the ANP, human's judgments are represented as exact numbers. However, in many practical cases the human preference model is uncertain and decision makers might be unable to assign exact numerical values to the comparison judgments. Since some of the evaluation criteria are subjective and qualitative in nature, it is very difficult for the decision-maker to express the preferences using exact numerical values and to provide exact pair-wise comparison judgments. It is more desirable for him to use interval or fuzzy evaluations [3]. To improve the ANP method, this paper discusses a fuzzy ANP approach using Gaussian fuzzy numbers to represent decision makers' comparison judgments and extent analysis method to decide the final priority of different decision criteria. The proposed model uses the linguistic

* Corresponding author. Tel.: +20 1111935336.
E-mail addresses: hehefny@ieee.org (H.A. Hefny), hamed_30@hotmail.com (H.M. Elsayed), hishamfaly@hotmail.com (H.F. Aly).

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variables and Gaussian fuzzy numbers as a pair-wise comparison scale for deriving the priorities of different selection attributes and sub-attributes. In the last step, the priority weights for main attributes, sub-attributes and alternatives are combined to determine the priority weights of the alternatives. The alternative with the highest priority weight is selected as the best alternative.

Erginel and Şenturk developed a fuzzy ANP model to rank for three Global Systems for Mobile Communications (GSMs) operators [4]. Yuksel and Dagdeviren used fuzzy ANP to demonstrate a process for quantitative Strengths, Weaknesses, Opportunities and Threats (SWOTs) analysis that can be performed even when there is dependence among strategic factors for a textile firm [5].

This paper aims to propose a fuzzy ANP decision-making support system that helps decision-makers of any authority in selecting the best alternatives among several offers. Such a kind of systems often requires highly experienced decision makers to consider vague and uncertain information.

Fuzzy set theory offers a possible means of managing these kinds of data or information. On the other hand, ANP offers a means for dealing with different preferences made to different decision alternatives. The remainder of the paper is organized as follows: In Section 2, an overview on fuzzy sets, linguistic variables, Analytic Hierarchy Process (AHP), and fuzzy AHP applications in literature are given. In Section 3, Analytic network process (ANP) is illustrated. FANP based approach is discussed in Section 4. Gaussian fuzzy ANP (GFANP) proposed model to select the best alternative is developed and the steps of each stage of the procedure are explained in detail in Section 5. In Section 6, results which are produced from the model are discussed, and the paper ends with concluding remarks in Section 7.

Egypt had installed generating capacity of 20 gigawatts (GW) as of 2010, with plans to add 25 GW of additional generating capacity by 2020. Around 90% of Egypt's electric generating capacity is thermal (natural gas), with the remaining 10% hydroelectric, mostly from the Aswan High Dam. All oil-fired plants have been converted to run on natural gas as their primary fuel. Egypt is also planning to build a part-solar power plant at Kureimat, which will have a total planned capacity of 150 MW. A Netherlands-funded project is building 60 MW of wind power units in the Suez Canal area. Egypt also has a 22-MW nuclear research reactor at Inshas in the Nile Delta, built by INVAP S.A. of Argentina, which began operation in 1997 [6].

2. Literature survey

2.1. Fuzzy sets

When fuzzy set theory was presented, researchers considered decision making as one of the most attractive application fields of that theory [7]. Fuzzy decision theories attempt to deal with the vagueness and no specificity inherent in human formulation of preferences, constraints, and goals [8].

A fuzzy set A in X is formally defined as follows [9]:

$$A = \{(x, \mu_A(x)) | x \varepsilon X\} \tag{1}$$

where X is the universe of discourse and $\mu_A(x)$ is the membership degree of the element x in A.

2.2. Linguistic variables

The conventional techniques for system analysis are intrinsically unsuitable for dealing with humanistic systems, whose behavior is strongly influenced by human judgment, perception, and emotions [10]. This is a manifestation of what might be called the principle of incompatibility: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance become almost mutually exclusive characteristics". Because of this belief Zadeh proposed the concept of linguistic variables as an alternative approach to modeling human thinking an approach that, in an approximate manner, serves to summarize information and express it in terms of fuzzy sets instead of crisp numbers [11].

2.3. The Analytic Hierarchy Process (AHP)

The AHP approach was developed in the early 1970s in response to military contingency planning, scarce resources allocation, and the need for political participation in disarmament agreements [12,13]. All these problems rely heavily on measurement and tradeoff of intangibles in a multi-criteria process. The AHP is a structured method to elicit preference opinion from decision makers. Its methodological procedure can easily be incorporated into multiple objective programming formulations with interactive solution process [12–14]. The AHP approach involves decomposing a complex and unstructured problem into a set of components organized in a multilevel hierarchic form [14]. A salient feature of the AHP is to quantify decision makers' subjective judgments by assigning corresponding numerical values based on the relative importance of factors under consideration. A conclusion can be reached by letting the judgments determine the overall priorities of variables [15]. The Analytic Hierarchy Process (AHP) finds out the "best" alternative out of several ones by considering a number of conflicting criteria. In the AHP one creates a hierarchy or network to represent a decision and establishes a matrix containing the pair wise comparison judgments for the elements linked under a parent element. The hierarchy is formed in a way that it enables the use of elements in a level to compare the elements in the level immediately below. A hierarchy should be rich enough to represent the problem, but simple enough to reflect sensitivity. Paired comparisons are essential. One first makes the paired comparisons, based on the preference table, and then derives the priorities from them. Paired comparisons are the engine for generating relative measurement. One then derives a priority vector of relative weights for these elements. There is one such matrix for every parent element. All the priority vectors are appropriately weighted and summed to obtain the overall priorities for the alternatives of a decision [16].

2.3.1. The standard AHP

Satty demonstrated mathematically that the eigenvector solution was the best approach to get a ranking of priorities from a pair wise matrix in the standard AHP [14]. Table 1 represents the standard preference table used by Saaty [17]. Table 2 represents a modified preference table that is used currently in several cases [18].

Table 1 The standard preference table of AHP.

Linguistic variable	Crisp number
Equally preferred (EP)	1
Equally to Moderately preferred (WMP)	2
Moderately preferred (MP)	3
Moderately to strongly preferred (MSP)	4
Strongly preferred (SP)	5
Strongly to very strongly preferred (SVP)	6
Very strongly preferred (VP)	7
Very strongly to extremely preferred (VEP)	8
Extremely preferred (XP)	9

Table 2 The modified preference table of AHP.

Linguistic variable	Crisp number
Equally preferred (EP)	1
Equally to Weakly preferred (EWP)	2
Weakly preferred (WP)	3
Weakly to Moderately preferred (WMP)	4
Moderately preferred (MP)	5
Moderately to strongly preferred (MSP)	6
Strongly preferred (SP)	7
Strongly to very strongly preferred (SVP)	8
Very strongly preferred (VP)	9
Very strongly to extremely preferred (VEP)	10
Extremely preferred (XP)	11

2.3.2. Extent fuzzy AHP (FAHP)

The most common method among different FAHP methods is the extent analysis method proposed by Chang [19]. The earliest work in fuzzy AHP compared fuzzy ratios described by triangular membership functions [20]. Cheng and Mon proposed a new algorithm for evaluating weapon systems by Analytical Hierarchy Process (AHP) based on fuzzy scales, which is a multiple criteria decision making approach in a fuzzy environment [21]. Cheng proposed a new algorithm for evaluating naval tactical missile systems by the fuzzy Analytical Hierarchy Process based on grade value of membership function [22]. This algorithm was applied to a missile system evaluation and selection problem. Kuo et al. developed a decision support system using fuzzy sets theory integrated with analytic hierarchy process for locating a new convenience store [23]. Altinoz examined supplier selection in general and specifically in the textile sector [24]. Kahraman et al. used fuzzy Analytic Hierarchy Process (AHP) to select the best supplier firm providing the most satisfaction for the criteria determined in the white good sector [25]. Chan and Kumar discussed a fuzzy AHP approach using triangular fuzzy numbers to represent decision makers' comparison judgments and fuzzy synthetic extent analysis method to decide the final priority of different decision criteria [3]. The main objective is the selection of best global supplier for a manufacturing firm. Haq and Kannan proposed a structured model for evaluating vendor selection using the Analytical Hierarchy Process (AHP) and fuzzy AHP [26]. The extent analysis method is used to consider the extent of an object to be satisfied for the goal, that is, satisfied extent. In the method, the ''extent'' is quantified by using a fuzzy number. On the basis of the fuzzy values for the extent analysis of each object, a fuzzy synthetic degree value can be obtained as follows:

Let $X = \{x_1, x_2, \ldots, x_n\}$ represents the elements of the alternatives as an object set, and let $U = \{u_1, u_2, \ldots, u_m\}$ represents the elements of the criteria as a goal set. Therefore, m extent analysis values for each object can be obtained with the following signs:

$$M_{g_i}^1, M_{g_i}^2, \ldots, M_{g_i}^m \quad i = 1, 2, \ldots, n \tag{2}$$

where all the $M_{g_i}^j$ $(j = 1, 2, \ldots, m)$ are triangular fuzzy numbers (TFNs).

The steps of Chang's extent analysis were given by Kahraman et al. [27] and Dağdeviren et al. [28].

2.4. Why ANP?

Although the AHP technique removes the deficiencies inherent in the measurement and evaluation steps of the problem analysis, it does not measure the possible dependencies among factors. The AHP method assumes that the factors presented in the hierarchical structure are independent; however, this is not always a reasonable presumption. The possible dependency among factors can only be determined as a result of internal and external environmental analyses.

3. Analytic Network Process (ANP) approach

ANP is a multi-attribute, decision-making approach based on the reasoning, knowledge, and experience of the experts in the field. ANP can act as a valuable aid for decision making involving both tangible as well as intangible attributes that are associated with the model under study. ANP relies on the process of eliciting managerial inputs, thus allowing for a structured communication among decision makers. Thus, it can act as a qualitative tool for strategic decision-making problems [29]. AHP method does not measure the possible dependencies among factors. It assumes that the factors presented in the hierarchical structure are independent; however, this is not always a reasonable assumption. The possible dependency among factors can only be determined as a result of internal and external environmental analyses. The ANP is a generalization of the AHP. While the AHP represents a framework with a uni-directional hierarchical AHP relationship, the ANP allows for complex interrelationships among decision levels and attributes [5]. For instance, not only does the importance of the criteria determine the importance of the alternatives, as in a hierarchy, but the importance of the alternatives may also have an impact on the importance of the criteria. Therefore, a hierarchical representation with a linear top-to-bottom structure is not suitable for a complex system. A system with feedback can be represented by a network.

4. Fuzzy ANP

The inability of ANP to deal with the impression in the pair wise comparison process has been improved in fuzzy ANP. Instead of a crisp value, fuzzy ANP applies a range of values to incorporate the decision maker's uncertainly. It enhances the potential of the ANP for dealing with imprecise and uncertain human comparison judgments. Ramik developed a decision system using ANP and fuzzy inputs [30]. In this paper extended arithmetic operations with fuzzy numbers are proposed as well as ordering fuzzy relations to compare fuzzy outcomes. Kaur and Mahanti developed a fuzzy ANP-based approach for selecting ERP vendors [31]. In this paper ANP equipped with fuzzy logic helps in overcoming the impreciseness and vagueness in the performance. Wu et al. developed a fuzzy ANP-based approach to evaluate medical organizational performance [32]. This paper proposes an evaluation model using fuzzy analytic network process (FANP). The proposed model can provide Taiwan's hospital accreditation policy a reference material, making it highly applicable for academic and government purposes. Rafiei and Rabbani developed an ordered partitioning in hybrid MTS/MTO contexts using fuzzy ANP [33]. In this paper, a model based on analytic network process is developed to tackle the addressed decision. Since the regarded decision deals with the uncertainty and ambiguity of data as well as experts' and managers' linguistic judgments, the proposed model is equipped with fuzzy sets theory. Rouyendegh and Erol developed the DEA – fuzzy ANP department ranking model applied in Iran Amirkabir University [34]. This research is a two-stage model designed to fully rank the organizational departments where each department has multiple inputs and outputs.

5. A Gaussian fuzzy ANP proposed model (GFANP)

5.1. Problem formulation

It is required to develop a decision-making system, which helps decision-makers in the Egyptian government to select the best alternatives for the different scenarios of electrical power generation in Egypt. The highest priority would be the best (see Fig. 1). There are three alternative scenarios:

Alt#1: the current one,
Alt#2: 20% nuclear, 75% petrol, 5% other, and

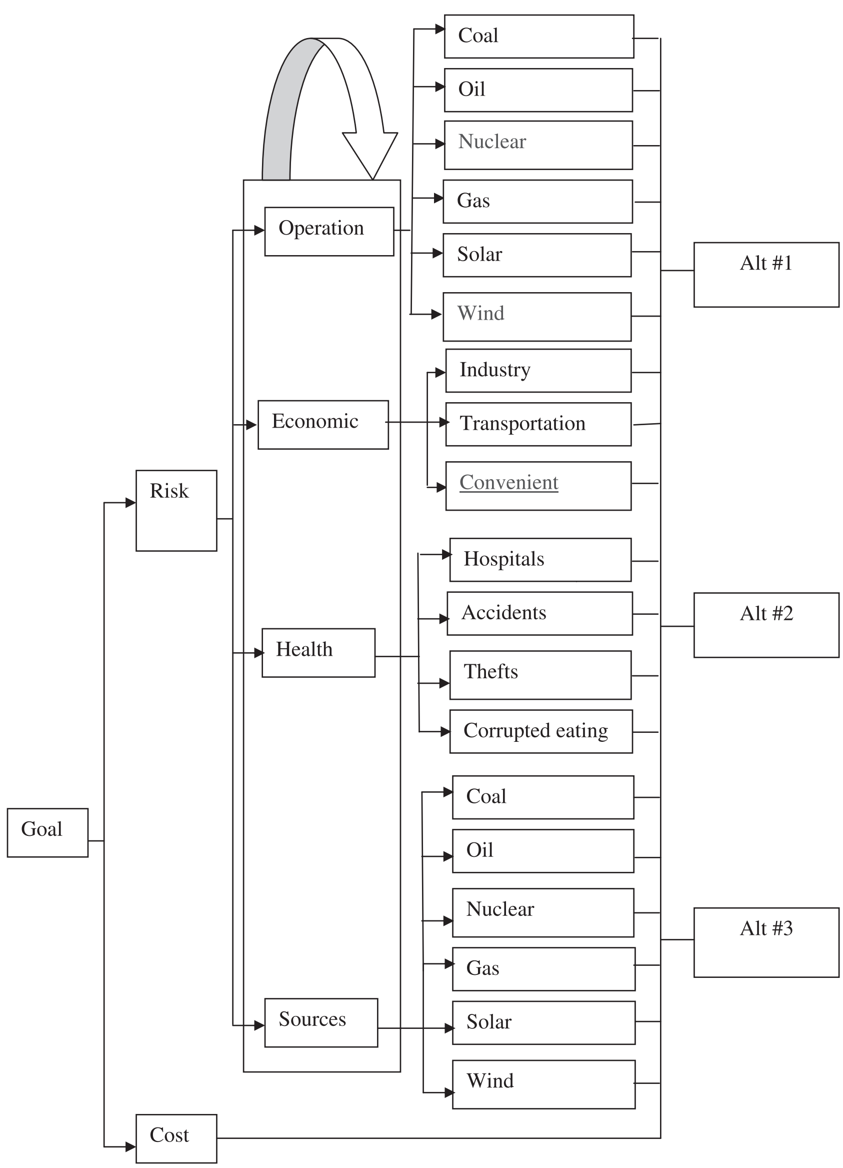

Figure 1 The ANP model.

Alt#3: 25% nuclear, 65% petrol, 5% solar, 5% other.

5.2. The problem of triangular fuzzy numbers

The following two cases illustrate the problem with triangular fuzzy numbers.

Case 1: It is required to rank the fuzzy numbers shown in Fig. 2 with the FANP methodology. Therefore we have the degree of possibility of $(M_1 = (l_1,m_1,u_1)) \geqslant (M_2 = (l_2,m_2,u_2))$ is defined as:

$$V(M_1 \geqslant M_2) = \mu(d_1) \quad (3)$$

And the degree of possibility of $(M_1 = (l_1,m_1,u_1)) \geqslant (M_3 = (l_3,m_3,u_3))$ is defined as

$$V(M_1 \geqslant M_3) = 0 \quad \text{as} \quad u_1 < l_3 \quad (4)$$

Assume that

$$d'(A_i) = \min V(S_i > S_k) \quad \text{for } k = 1,2,\ldots,n; \ k \neq i. \quad \text{Then} (5)$$

$$d'(A_1) = \min(\mu(d_1),0) = 0 \quad (6)$$

The weight vector is given by

$$W' = (d'(A_1), d'(A_2), d'(A_3))^T \quad (7)$$

$$\text{Then } W' = (0, \mu(d_2), 1) \quad (8)$$

Via normalization, the normalized weight vector is

$$W = (0, w_2, w_3) \quad \text{where } w_2 \text{ and } w_3 \text{ are nonzero values.} \quad (9)$$

Then by using triangular fuzzy ANP, the first item is completely eliminated and its weight over others will be zero.

If there are i items and $u_1 < l_2, \ldots, l_i$ then the same case will be found and

$$W = (0, w_2, w_3, \ldots, w_n)^T \quad \text{where } w_2, w_3, \ldots, w_n \quad (10)$$

are non zero numbers.

It is possible to have more than one item having weights equal to zero. In such a case more than one fuzzy number will be ranked equally. From the perspective of FANP, this means that some alternatives will be wrongly considered equivalent.

Case 2: In case 1, each fuzzy number intersects at least with one fuzzy number. In this case, we assume that some fuzzy numbers do not intersect at all, as shown in Fig. 3.

The degree of possibility of $(M_1 = (l_1,m_1,u_1)) \geqslant (M_2 = (l_2,m_2,u_2))$ is defined as

$$V(M_1 \geqslant M_2) = \mu(d_1) \quad (11)$$

And

The degree of possibility of $(M_1 = (l_1,m_1,u_1)) \geqslant (M_3 = (l_3,m_3,u_3))$ is defined as

$$V(M_1 \geqslant M_3) = 0 \text{ as } u_1 < l_3 \quad (12)$$

$$\text{Then } d'(A_1) = \min(\mu(d_1),0) = 0 \quad (13)$$

The degree of possibility of $(M_2 = (l_2,m_2,u_2)) \geqslant (M_1 = (l_1,m_1,u_1))$ is defined as

$$V(M_2 \geqslant M_1) = 1 \quad (14)$$

And

The degree of possibility of $(M_2 = (l_2,m_2,u_2)) \geqslant (M_3 = (l_3,m_3,u_3))$ is defined as

$$V(M_2 \geqslant M_3) = 0 \text{ as } u_2 < l_3 \quad (15)$$

$$\text{Then } d'(A_2) = \min(1,0) = 0 \quad (16)$$

The weight vector is given by

$$W' = (d'(A_1), d'(A_2), d'(A_3))^T \quad (17)$$

$$\text{Then } W' = (0,0,1)^T \quad (18)$$

Via normalization, the normalized weight vector is

$$W = (0,0,1)^T \quad (19)$$

If there are i items and $l_i > u_1, u_2, \ldots, u_{i-1}$ then the same case will be found and

$$W = (0,0,\ldots,1)^T. \quad (20)$$

Then by using triangular fuzzy ANP, only one item that has a weight equals to 1, while all other items are wrongly weighted as 0.

Thus, from the above discussion, it is clear that triangular fuzzy numbers, and even trapezoidal ones, have serious shortage when used as preference values.

5.3. The proposed model

The proposed model to overcome the problem of triangular fuzzy numbers depends on replacing them by Gaussian fuzzy numbers. It is clear that defining Gaussian fuzzy numbers over the preference scale results in real intersection between any

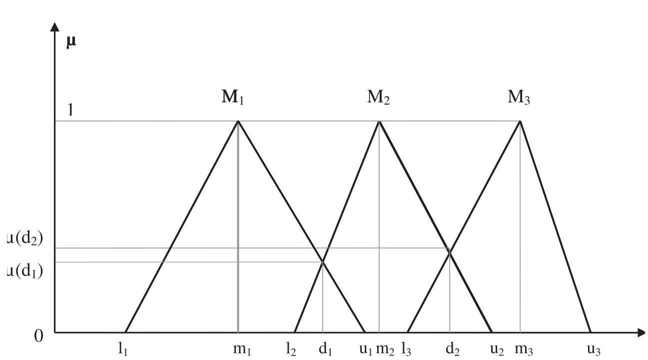

Figure 2 The fuzzy numbers need to be ranked (case 1).

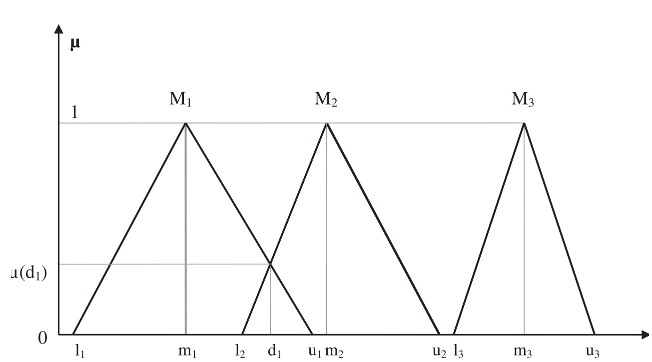

Figure 3 The fuzzy numbers need to be ranked (case 2).

Table 3 The preference table: μ = crisp number, $\sigma = 0.5$.

Linguistic variable	Crisp no.	Triang(x, a, b, c)	Gaussian(x, μ, σ)
Equally preferred (EP)	1	Triang$(x, 1, 1, 1)$	Gaussian$(x, 1, 0.5)$
Equally to Weakly preferred (EWP)	2	Triang$(x, 1.5, 2, 2.5)$	Gaussian$(x, 2, 0.5)$
Weakly preferred (WP)	3	Triang$(x, 2.5, 3, 3.5)$	Gaussian$(x, 3, 0.5)$
Weakly to Moderately preferred (WMP)	4	Triang$(x, 3.5, 4, 4.5)$	Gaussian$(x, 4, 0.5)$
Moderately preferred (MP)	5	Triang$(x, 4.5, 5, 5.5)$	Gaussian$(x, 5, 0.5)$
Moderately to strongly preferred (MSP)	6	Triang$(x, 5.5, 6, 6.5)$	Gaussian$(x, 6, 0.5)$
Strongly preferred (SP)	7	Triang$(x, 6.5, 7, 7.5)$	Gaussian$(x, 7, 0.5)$
Strongly to very strongly preferred (SVP)	8	Triang$(x, 7.5, 8, 8.5)$	Gaussian$(x, 8, 0.5)$
Very strongly preferred (VP)	9	Triang$(x, 8.5, 9, 9.5)$	Gaussian$(x, 9, 0.5)$
Very strongly to extremely preferred (VEP)	10	Triang$(x, 9.5, 10, 10.5)$	Gaussian$(x, 10, 0.5)$
Extremely preferred (XP)	11	Triang$(x, 10.5, 11, 11.5)$	Gaussian$(x, 11, 0.5)$

number and all the other numbers. This eliminates the problem of getting some alternatives have the same rank and consequently be treated equivalently. Thus, after illustrating that idea, we propose a modified preference table, shown in Table 3, in which we introduce Gaussian fuzzy numbers to the eleven-point scale table. It should be noted that the centers (μ's) of the Gaussian preference values must be the same as the crisp preference scale values. However, the widths (σ's) can be freely assumed according to the existing amount of uncertainty.

Gaussian functions have the advantage of being fully determined using only two parameters, i.e. center (μ) and width (σ) and its value never equals to zero (within the range of the preference scale values). Thus, the intersection must be existed between every fuzzy number and all the others. In this case, shortages of the triangular fuzzy number are overcome.

The definition of Gaussian function is as follows:

$$\text{Gaussian}(x : \mu, \sigma) = \exp\left[\frac{-(x-\mu)^2}{\sigma^2}\right] \tag{21}$$

At any level α, as in Fig. 4, it is shown that:

$$\alpha = \exp\left[\frac{-(x-\mu)^2}{\sigma^2}\right] \tag{22}$$

$$x_1 = \mu - \sigma \times \sqrt{-Ln(\alpha)} \text{ and} \tag{23}$$

$$x_2 = \mu + \sigma \times \sqrt{-Ln(\alpha)} \tag{24}$$

It is clear that as long as α level is small enough, then it is possible to get a good fuzzy triangular approximation of $G(x, \mu, \sigma)$ by $T(x, x_1, \mu, x_2)$. Such an approximation is useful for performing the fuzzy arithmetic operations to get S_i as shown in Eq. (25). Once, we get S_i's as triangle fuzzy numbers, they can be returned back to Gaussian to perform the ranking step.

For example suppose that

$\sigma = 0.5$ and $\alpha = 0.1$ then
$x_1 = \mu - 0.76$ and
$x_2 = \mu + 0.76$

The steps of the modified fuzzy ANP (FANP) method are illustrated as follows:

Let G_{ij} be the elements of the preference matrix after performing the triangular approximation, then:

Step 1:

$$\mathbf{S}_i = \frac{\sum_j G_{ij}}{\sum_i \sum_j G_{ij}} \tag{25}$$

$$= \frac{\sum_j (l_i^j, m_i^j, u_i^j)}{\sum_i \sum_j (l_i^j, m_i^j, u_i^j)} \tag{26}$$

where

$$l_i^j \cong m_i^j - \sigma_i^j \sqrt{-Ln(\alpha)} \tag{27}$$

$$u_i^j \cong m_i^j + \sigma_i^j \sqrt{-Ln(\alpha)} \tag{28}$$

To get good triangular approximation, we choose a low level for α.

For example let $\alpha = 0.001$

$$S_i = \frac{\left(\sum_j l_i^j, \sum_j m_i^j, \sum_j u_i^j\right)}{\left(\sum_i \sum_j l_i^j, \sum_i \sum_j m_i^j, \sum_i \sum_j u_i^j\right)} \tag{29}$$

$$= \left(\frac{\sum_j l_i^j}{\sum_i \sum_j u_i^j}, \frac{\sum_j m_i^j}{\sum_i \sum_j m_i^j}, \frac{\sum_j u_i^j}{\sum_i \sum_j l_i^j}\right) \tag{30}$$

$$\sum_j l_i^j = \sum_j m_i^j - \sum_j \sigma_i^j (\sqrt{-Ln(\alpha)}) \tag{31}$$

$$\sum_j u_i^j = \sum_j m_i^j + \sum_j \sigma_i^j (\sqrt{-Ln(\alpha)}) \tag{32}$$

$$\sum_i \sum_j l_i^j = \sum_i \sum_j m_i^j - \sum_i \sum_j \sigma_i^j \left(\sqrt{-Ln(\alpha)}\right) \tag{33}$$

$$\sum_i \sum_j u_i^j = \sum_i \sum_j m_i^j + \sum_i \sum_j \sigma_i^j (\sqrt{-Ln(\alpha)}) \tag{34}$$

$$\therefore S_i = \left(x_{s_i}^L, m_{s_i}, x_{s_i}^R\right)$$

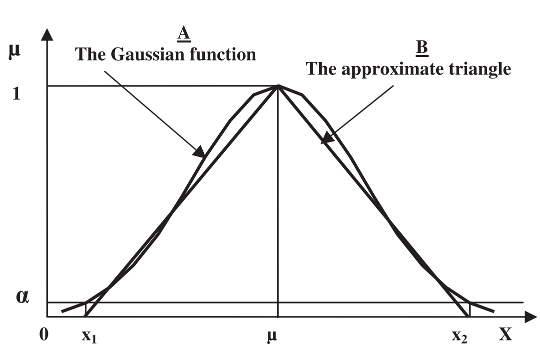

Figure 4 Gaussian function A and its approximated triangle B.

where

$$m_{s_i} = \frac{\sum_j m_i^j}{\sum_i \sum_j m_i^j} \tag{35}$$

$$x_{s_i}^L = \frac{\sum_j l_i^j}{\sum_i \sum_j u_i^j} \tag{36}$$

$$x_{s_i}^R = \frac{\sum_j u_i^j}{\sum_i \sum_j l_i^j} \tag{37}$$

Now, S_i can be returned back to a Gaussian fuzzy number (but asymmetric in this case) as follows:

$$\sigma_{s_i}^L = \frac{m_{s_i} - x_{s_i}^L}{\sqrt{-Ln(\alpha)}} \tag{38}$$

$$\sigma_{s_i}^R = \frac{x_{s_i}^R - m_{s_i}}{\sqrt{-Ln(\alpha)}} \tag{39}$$

where $\sigma_{s_i}^L$ is the width of the left branch of the Gaussian fuzzy number and $\sigma_{s_i}^R$ is the width of the right branch of the Gaussian fuzzy number.

Now, S_i becomes an asymmetric Gaussian number as follows:

$$\mu_{s_i}(x) = \left\{ \begin{array}{ll} \exp\left[-\left(\frac{x-m_{s_i}}{\sigma_{s_i}^L}\right)^2\right] & \text{if } x \leqslant m_{s_i} \\ \exp\left[-\left(\frac{x-m_{s_i}}{\sigma_{s_i}^R}\right)^2\right] & \text{if } x > m_{s_i} \end{array} \right\} \tag{40}$$

Step 2:

Let $\mu_1(x)$ and $\mu_2(x)$ be two Gaussian fuzzy numbers having the following forms:

$$\mu_{s_1}(x) = \left\{ \begin{array}{ll} \exp\left[-\left(\frac{x-m_{s_1}}{\sigma_{s_1}^L}\right)^2\right] & \text{if } x \leqslant m_{s_1} \\ \exp\left[-\left(\frac{x-m_{s_1}}{\sigma_{s_1}^R}\right)^2\right] & \text{if } x > m_{s_1} \end{array} \right\}, \tag{41}$$

and

$$\mu_{s_2}(x) = \left\{ \begin{array}{ll} \exp\left[-\left(\frac{x-m_{s_2}}{\sigma_{s_2}^L}\right)^2\right] & \text{if } x \leqslant m_{s_2} \\ \exp\left[-\left(\frac{x-m_{s_2}}{\sigma_{s_2}^R}\right)^2\right] & \text{if } x > m_{s_2} \end{array} \right\} \tag{42}$$

The intersection point between two Gaussian functions is shown in Fig. 5.

$$\boldsymbol{v} = \left\{ \begin{array}{ll} \exp\left[-\left(\frac{(m_{s_2}-m_{s_1})}{(\sigma_{s_1}^L+\sigma_{s_2}^R)}\right)^2\right] & \text{if } m_{s_1} > m_{s_2} \\ \exp\left[-\left(\frac{(m_{s_2}-m_{s_1})}{(\sigma_{s_1}^R+\sigma_{s_2}^L)}\right)^2\right] & \text{if } m_{s_1} < m_{s_2} \end{array} \right\} [35]. \tag{43}$$

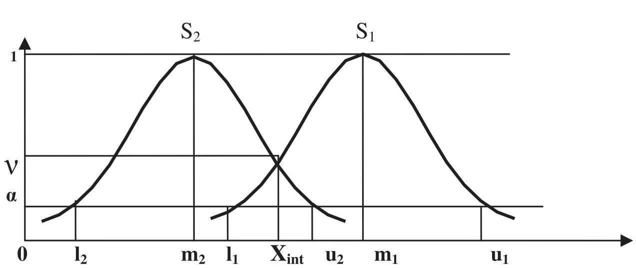

Figure 5 Intersection point between two Gaussian functions.

The degree of possibility of $S_2 = \mu_{S_2}(x) \geqslant S_1 = \mu_{S_1}(x)$ is defined as

$$V(S_2 \geqslant S_1) = \text{hgt}(S_1 \cap S_2) = \mu_{S_2}(x_{\text{int}}) \tag{44}$$

$$V(S_2 \geqslant S_1) = \left\{ \begin{array}{ll} 1 & \text{if } m_{s_2} \geqslant m_{s_1}, \\ \exp\left[-\left(\frac{(m_{s_2}-m_{s_1})}{(\sigma_{s_2}^R+\sigma_{s_1}^L)}\right)^2\right] & \text{if } m_{s_2} < m_{s_1} \end{array} \right\} \tag{45}$$

where $\mathbf{X_{int}}$ is the ordinate of the inner intersection point between $\mu_{S_2}(x)$ and $\mu_{S_1}(x)$. To compare S_1 and S_2, the values of both $V(S_2 \geqslant S_1)$ and $V(S_1 \geqslant S_2)$ are needed.

Step 3:

The degree of possibility for a Gaussian fuzzy number S_i to be greater than k Gaussian fuzzy numbers $S_i(i = 1, 2, \ldots, k)$ can be defined by

$$V(S > S_1, S_2, \ldots, S_k) = V[(S > S_1) \text{ and } (S > S_2) \text{ and } \cdots \text{ and } (S > S_k)] = \min V(S > S_i), \quad i = 1, 2, 3, \ldots, k. \tag{46}$$

Assume that

$$d'(A_i) = \min V(S_i > S_j) \quad \text{for } j = 1, 2, \ldots, n; \quad j \neq i. \tag{47}$$

Then the weight vector is given by:

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T, \tag{48}$$

where A_i $(i = 1, 2, \ldots, n)$ are n elements.

Step 4:

Via normalization, the normalized weight vector is

$$W = (d(A_1), d(A_2), \ldots, d(A_n))^T, \tag{49}$$

$$\text{where } d(A_i) = \frac{d'(A_i)}{\sum_i d'(A_i)} \tag{50}$$

This gives the required priority weights of one alternative among others.

6. Experimental results and discussion

It is needed to rank the alternatives A_1, A_2, A_3, A_4, A_5 over Criteria C_1, C_2, C_3, C_4. According to Fig. 1, the linguistic preference matrices of the different criteria nodes are given in Tables 4–9. On the other hand, the inner dependences matrices with respect to different criteria nodes are given in Table 10. The inner dependence among factors is shown in Fig. 6.

$$\mathbf{w}_1 = \begin{bmatrix} \text{Operation} \\ \text{Economic} \\ \text{Health} \\ \text{Sources} \end{bmatrix} = \begin{bmatrix} 0.30 \\ 0.27 \\ 0.23 \\ 0.20 \end{bmatrix} \tag{51}$$

Table 4 The evaluation matrix with respect to the Goal.

	Risk	Cost
Risk	EP	WMP
Cost		EP

Table 5 The evaluation matrix with respect to the Risk.

	Operation	Economic	Health	Sources
Operation	EP	WMP	EWP	WP
Economic		EP	EWP	EWP
Health			EP	EP
Sources				EP

Table 6 The evaluation matrix with respect to the Operation Risk.

	Coal	Oil	Nuclear	Gas	Solar	Wind
Coal	EP					
Oil	WMP	EP	EP			
Nuclear	EWP		EP			
Gas	WP	EWP	EWP	EP		
Solar	MP	WP	EWP	EWP	EP	
Wind	WMP	WP	EWP	EWP	EWP	EP

Table 7 The evaluation matrix with respect to the Economic Risk.

	Industry	Transportation	Convenient
Industry	EP	EP	EP
Transportation		EP	EP
Convenient			EP

Table 8 The evaluation matrix with respect to the Health Risk.

	Hospitals	Accidents	Thefts	Corr. Eating
Hospitals	EP	WP	EWP	EWP
Accidents		EP	WP	EWP
Thefts			EP	EP
Corr. eating				EP

Table 9 The evaluation matrix with respect to the sources risk.

	Coal	Oil	Nuclear	Gas	Solar	Wind
Coal	EP					
Oil	EWP	EP				
Nuclear	WP	EWP	EP			
Gas	WP	EWP	EWP	EP		
Solar	MP	WMP	WP	EWP	EP	EP
Wind	MP	WMP	WP	EWP		EP

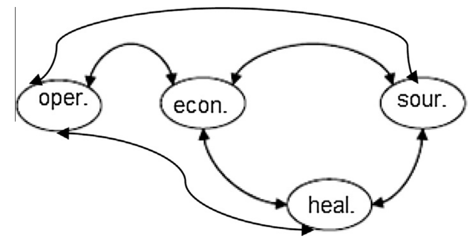

Figure 6 Inner dependence among factors.

Table 11 The evaluation matrix with respect to the Cost and the Coal.

	Cost			Coal		
	Alt. #1	Alt. #2	Alt. #3	Alt. #1	Alt. #2	Alt. #3
Alt. #1	EP	EWP	WP	EP		
Alt. #2		EP		EWP	EP	
Alt. #3		EWP	EP	WP	EWP	EP

6.1. Inner relationship

$$W_2 = \begin{bmatrix} 1.00 & 0.33 & 0.33 & 0.33 \\ 0.25 & 1.00 & 0.48 & 0.48 \\ 0.25 & 0.48 & 1.00 & 0.19 \\ 0.50 & 0.19 & 0.19 & 1.00 \end{bmatrix} \tag{52}$$

$$V_{\text{factors}} = W_2 * w_1 = \begin{bmatrix} 1.00 & 0.33 & 0.33 & 0.33 \\ 0.25 & 1.00 & 0.48 & 0.48 \\ 0.25 & 0.48 & 1.00 & 0.19 \\ 0.50 & 0.19 & 0.19 & 1.00 \end{bmatrix} * \begin{bmatrix} 0.30 \\ 0.27 \\ 0.23 \\ 0.20 \end{bmatrix} = \begin{bmatrix} 0.27 \\ 0.28 \\ 0.23 \\ 0.22 \end{bmatrix} \tag{53}$$

Sample of the pair-wise comparisons between alternatives Alt.#1, Alt.#2, and Alt.#3 over criteria are given in Table 11.

6.2. Alternatives

Then, the overall normalized priority weight vector of the alternatives is obtained as follows:

$$W = [0.25, 0.33, 0.42]^T, \tag{54}$$

which means that Alt.#3 is the best alternative. Therefore 25% of the generated electricity is come from nuclear power stations, 65% from petrol thermal stations, 5% from solar stations and 5% from other recourses.

Table 10 The inner dependences matrix.

Operation (Op.)				Economic (Ec.)				Health (Hel.)				Sources (Src.)			
	Ec.	Hel.	Src.		Op.	Hel.	Src.		Op.	Ec.	Src.		Op.	Ec.	Hel.
Ec.	EP	EP		Op.	EP		EWP	**Op.**	EP		EWP	Op.	EP		EWP
Hel.		EP		Hel.	EWP	EP	EWP	Ec.	EWP	EP	EWP	Ec.	EWP	EP	EWP
Src.	EWP	EWP	EP	Src.			EP	Src.			EP	Hel.			EP

7. Conclusions

In the proposed model it is possible to benefit from the advantages of both interval and fixed value judgments. Shortages caused by each of them can be avoided. GFANP model provides expert judgments the flexibility of using interval values in their preference matrices instead of crisp values. Gaussian fuzzy numbers are used instead of triangular numbers. By using them the case of zero weights will never exist. We recommend decision-makers in the Egyptian government to build more nuclear power stations to cover 25% of the generated electricity in Egypt. We also recommend them to construct solar power stations to cover 5% of the generated electricity.

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